Advanced Mathematics Support Programme®

Did you know ?

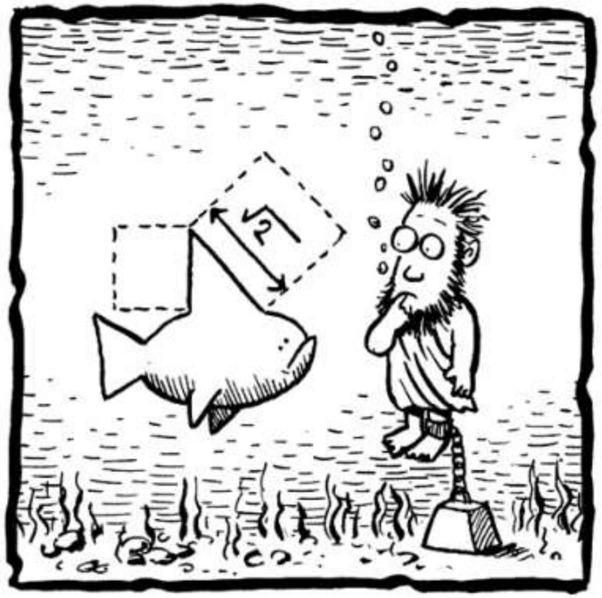
Maths can be murderous!

You will have heard of Pythagoras and his theorem but have you heard of Hippasus who was one of his followers?

Pythagoreans preached that all numbers could be expressed as the ratio of integers – i.e. fractions.

Hippasus is sometimes credited with the discovery of the existence of irrational numbers – proving it for $\sqrt{2}$.

Following which, he was drowned at sea!



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Surds 1

1. Simplify $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

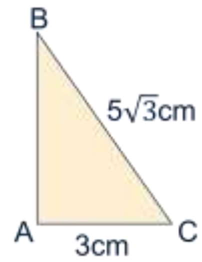
2. Simplify $\sqrt{2} \times \sqrt{6}$

3. Simplify fully $(4\sqrt{3})^2$

4. Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$

5. Calculate $\frac{\sqrt{54}}{\sqrt{6}}$

6. Rationalise the denominator of $\frac{4}{\sqrt{3}}$

7. 

Find the length AB

8. A rectangle has an area of $8\sqrt{15}$ cm^2 and a length of $2\sqrt{3}$ cm.

   Find the width of the rectangle

Surds 1

Solutions on the next slide….

Surds 1 Solutions

1. Simplify $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

2. Simplify $\sqrt{2} \times \sqrt{6}$

3. Simplify fully $(4\sqrt{3})^2$

4. Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$

1. $= 8\sqrt{a}$

Here we are just using the same skills here as when collecting like terms with algebraic expressions e.g. $x + 2x + 5x = 8x$

2. $= \sqrt{2 \times 6} = \sqrt{12}$

$= \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

3. $= 4\sqrt{3} \times 4\sqrt{3}$

$= 4 \times 4 \times \sqrt{3} \times \sqrt{3}$

$= 16 \times 3$

$= 48$

4. $\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

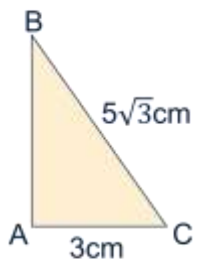
$3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

Unsure about any of these? Search Simplifying surds Then try Skills check 2 to reassess yourself.

Surds 1 Solutions

5. Calculate $\dfrac{\sqrt{54}}{\sqrt{6}}$

6. Rationalise the denominator of $\dfrac{4}{\sqrt{3}}$

7. 

Find the length AB

8. A rectangle has an area of $8\sqrt{15}$ cm^2 and a length of $2\sqrt{3}$ cm.

Find the width of the rectangle

→ 5. $$\frac{\sqrt{54}}{\sqrt{6}} = \frac{\sqrt{9 \times 6}}{\sqrt{6}} = \frac{\sqrt{9}\times\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{\sqrt{6}} = 3$$

→ 6. $$\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

→ 7. Using Pythagoras' theorem

$$AB^2 = \left(5\sqrt{3}\,\right)^2 - 3^2$$
$$AB^2 = (25 \times 3) - 9$$
$$AB^2 = 66 \;\; so \;\; AB = \sqrt{66}\text{ cm}$$

→ 8. $$8\sqrt{15} = \text{width} \times 2\sqrt{3}$$
$$8\sqrt{15} \div 2\sqrt{3} = \text{width}$$
$$\frac{8\sqrt{15}}{2\sqrt{3}} = \frac{8 \times \sqrt{5} \times \sqrt{3}}{2\sqrt{3}} = 4\sqrt{5}\text{cm}$$

Unsure about any of these? Search Simplifying surds and rationalising the denominator
Then try Skills check 2 to reassess yourself.

Surds 2

1. Simplify $\sqrt{d}$ + $6\sqrt{d}$ - 3 $\sqrt{d}$

2. Simplify $2\sqrt{b}$ x $4\sqrt{3}$

3. Simplify fully ($4\sqrt{5}$) ^{2}

4. Write $\sqrt{75}$ + $\sqrt{48}$ - 2 $\sqrt{12}$ in the form k $\sqrt{3}$

5. Simplify $\dfrac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$

6. Rationalise the denominator of $\dfrac{2\sqrt{2}}{\sqrt{5}}$

7. Evaluate $\dfrac{1}{\sqrt{2}} + \dfrac{\sqrt{3}}{\sqrt{6}}$

   Give you answer in simplest form.
   Rationalise the denominator.

8. A triangle has a base of 3 $\sqrt{2}$ and a perpendicular height of 5 $\sqrt{8}$.
   Calculate the area of the triangle.

Surds 2

Solutions on the next slide….

Surds 2 Solutions

1. Simplify $\sqrt{d}$ + $6\sqrt{d}$ - 3 $\sqrt{d}$ → $= 4\sqrt{d}$

2. Simplify $2\sqrt{b}$ x $4\sqrt{3}$ →

$$= 2 \times 4 \times \sqrt{b} \times \sqrt{3}$$
$$= 8 \times \sqrt{b \times 3}$$
$$= 8\sqrt{3b}$$

3. Simplify fully ($4\sqrt{5}$) ^{2} →

$$= 4\sqrt{5} \times 4\sqrt{5}$$
$$= 4 \times 4 \times \sqrt{5} \times \sqrt{5}$$
$$= 16 \times 5$$
$$= 80$$

4. Write $\sqrt{75}$ + $\sqrt{48}$ - 2 $\sqrt{12}$ in the form k $\sqrt{3}$ →

$$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$
$$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$$
$$2\sqrt{12} = 2 \times \sqrt{4} \times \sqrt{3} = 2 \times 2\sqrt{3} = 4\sqrt{3}$$

$$5\sqrt{3} + 4\sqrt{3} - 4\sqrt{3} = 5\sqrt{3}$$

Surds 2 Solutions

5. Simplify $\dfrac{\sqrt{125}-2\sqrt{20}}{\sqrt{5}}$

$$=\frac{\sqrt{25}\sqrt{5}-2\sqrt{4}\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{5}-2\times2\sqrt{5}}{\sqrt{5}}$$
$$=\frac{5\sqrt{5}-4\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{\sqrt{5}}=1$$

6. Rationalise the denominator of $\dfrac{2\sqrt{2}}{\sqrt{5}}$

$$\frac{2\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{2}\sqrt{5}}{\sqrt{5}\sqrt{5}}=\frac{2\sqrt{10}}{5}$$

7. Evaluate $\dfrac{1}{\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{6}}$

Give you answer in simplest form.
Rationalise the denominator.

Need a common denominator to add fractions

$$=\frac{1\ (\times\sqrt{3})}{\sqrt{2}\ (\times\sqrt{3})}+\frac{\sqrt{3}}{\sqrt{6}}=\frac{\sqrt{3}}{\sqrt{6}}+\frac{\sqrt{3}}{\sqrt{6}}=\frac{2\sqrt{3}}{\sqrt{6}}$$
$$=\frac{2\sqrt{3}}{\sqrt{2}\sqrt{3}}=\frac{2\ (\times\sqrt{2})}{\sqrt{2}\ (\times\sqrt{2})}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

8. A triangle has a base of 3 $\sqrt{2}$ and a perpendicular height of 5 $\sqrt{8}$. Calculate the area of the triangle.

$$\text{Area}\ =\frac{1}{2}\times3\sqrt{2}\times5\sqrt{8}=\frac{1}{2}\times3\times5\times\sqrt{2}\sqrt{8}$$
$$=\frac{1}{2}\times15\times\sqrt{16}=\frac{1}{2}\times15\times4$$
$$=30\ \text{cm}^2$$

These are just example solutions and there is more than one way to do many of these questions – some are more efficient methods than others!

Another brick in the wall

Complete the empty boxes in the pyramid.
Each box is the sum of the two boxes directly below it.

$31\sqrt{3} + 18\sqrt{5}$

$\sqrt{75}$	$2\sqrt{3}$	$\sqrt{245}$	$-\sqrt{180}$	$2\sqrt{243}$

*Hint: You may need to simplify some of the surds in the bottom row to get started.

Another brick in the wall

Solutions on the next slide….

Another brick in the wall Solutions

Complete the empty boxes in the pyramid.
Each box is the sum of the two boxes directly below it.

$31\sqrt{3} + 18\sqrt{5}$

$11\sqrt{3} + 15\sqrt{5}$ | $20\sqrt{3} + 3\sqrt{5}$

$9\sqrt{3} + 7\sqrt{5}$ | $2\sqrt{3} + 8\sqrt{5}$ | $18\sqrt{3} - 5\sqrt{5}$

$7\sqrt{3}$ | $2\sqrt{3} + 7\sqrt{5}$ | $\sqrt{5}$ | $-6\sqrt{5} + 18\sqrt{3}$

$\sqrt{75}$ | $2\sqrt{3}$ | $\sqrt{245}$ | $-\sqrt{180}$ | $2\sqrt{243}$

True or False?

Decide if each of the following expressions is True or False

1. $\sqrt{9} + \sqrt{4} = \sqrt{13}$

2. $\sqrt{a} \times \sqrt{b} = \sqrt{c}$

3. $\sqrt{(8)^2} = 8$

4. $10\sqrt{2} = 5\sqrt{8}$

5. $\dfrac{\sqrt{12} \times \sqrt{3}}{\sqrt{9}} = 2$

6. $\sqrt{2}^3 = 2\sqrt{2}$

7. $\sqrt{ab}^2 = ab$

8. $2\sqrt{100} = \sqrt{200}$

Are there any statements that are sometimes true but not always true?

Can you explain why?

True or False?

Solutions on the next slide….

True or False? Solutions

Decide if each of the following expressions is True or False

1. $\sqrt{9}+\sqrt{4}=\sqrt{13}$ False → $\sqrt{9}=3\ and\ \sqrt{4}=2$

   $3+2=5\ and\ \sqrt{13}\neq 5$

2. $\sqrt{a}\times\sqrt{b}=\sqrt{c}$

   True → when a x b = c e.g. a=5, b=6, c=30

   False → otherwise

3. $\sqrt{(8)^2}=8$ True → $\sqrt{(8)^2}=\sqrt{64}=8$

4. $10\sqrt{2}=5\sqrt{8}$ True → $10\sqrt{2}=\sqrt{100}\times\sqrt{2}=\sqrt{200}$

   $5\sqrt{8}=\sqrt{25}\times\sqrt{8}=\sqrt{200}$

True or False? Solutions

Decide if each of the following expressions is True or False

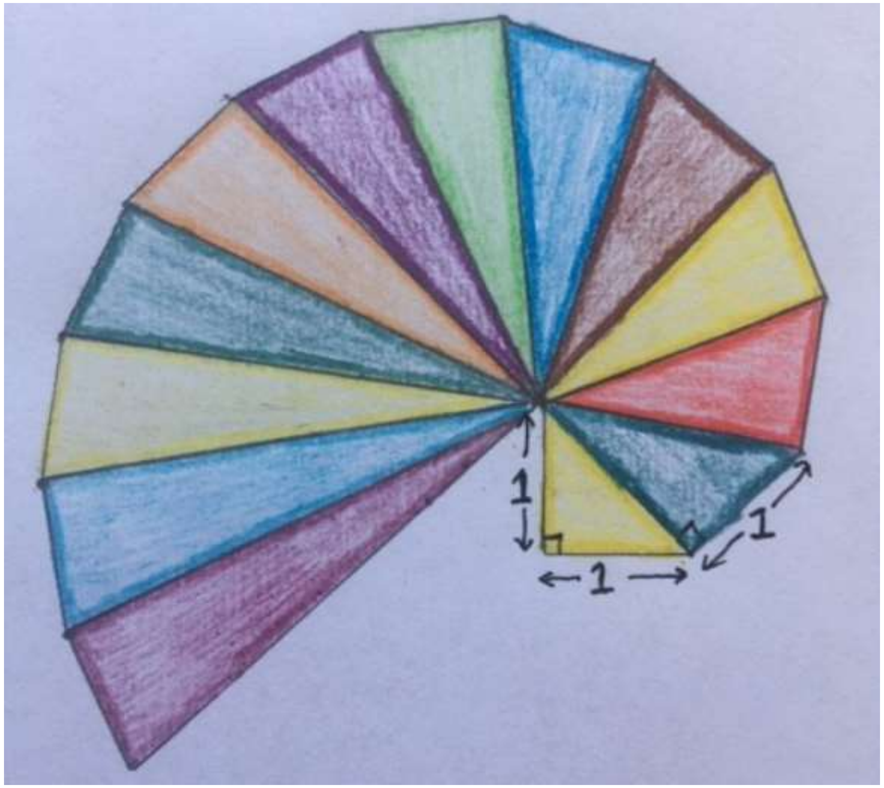
5. $\dfrac{\sqrt{12}\times\sqrt{3}}{\sqrt{9}} = 2$ True → $\dfrac{\sqrt{12}\times\sqrt{3}}{\sqrt{9}} = \dfrac{\sqrt{36}}{\sqrt{9}} = \sqrt{\dfrac{36}{9}} = \sqrt{4}$

6. $\sqrt{2}^{3} = 2\sqrt{2}$ True → $\sqrt{2}^{3} = \sqrt{2}\times\sqrt{2}\times\sqrt{2}$
$= \sqrt{4}\times\sqrt{2} = 2\sqrt{2}$

7. $\sqrt{ab}^{2} = ab$ True → $\sqrt{ab}^{2} = \sqrt{ab}\times\sqrt{ab} = ab$

8. $2\sqrt{100} = \sqrt{200}$ False → $2\sqrt{100} = \sqrt{4}\times\sqrt{100} = \sqrt{400}$

Wheel of Theodorus



The diagram shows a spiral made up of right-angled triangles.

The shortest side of each triangle measures 1 unit.

Can you see how it is constructed?

Find the length of the hypotenuse for the first few triangles.

What do you notice?

Which triangles would have a side of length 3?

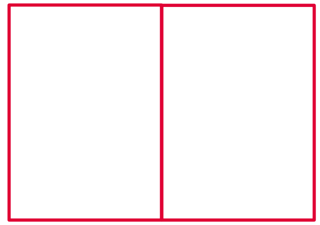
What other questions could you ask about the diagram?

Wheel of Theodorus

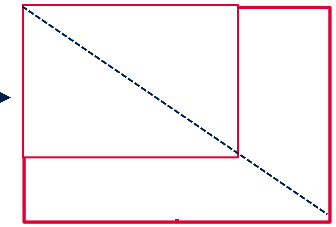
Follow the link to the solutions

Take a sheet of A4 paper

Actually take two sheets and place them side by side to make a piece of A3.

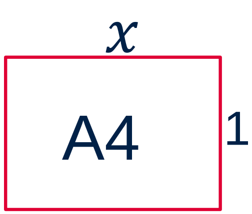


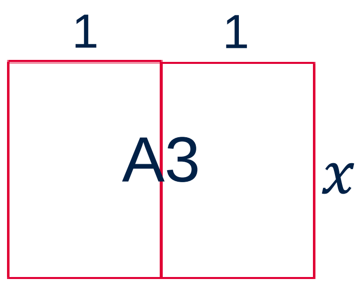
Then place another sheet of A4 on top of it like this:



What do you notice about the ratio of the sides of an A3 sheet compared with the A4 sheet?

Thinking about the ratio of the long side to the short side we get:





$$\frac{x}{1} = \frac{2}{x} \rightarrow x^2 = 2 \rightarrow x = \sqrt{2}$$

Therefore, for A4, A3, A2, etc… the length of the long side divided by the length of the short side is always $\sqrt{2}$

For more questions and to explore A4 sized paper further try this activity

Take a sheet of A4 paper

Follow the link to the solutions

Still want more?

Read about how Irrational numbers can "Inspir-al" you!
It's where mathematics and art meet!

Discover the proof, that $\sqrt{2}$ is irrational – without getting murdered like Hippasus.

Watch this video to find out more about the special properties of A4 paper and discover what makes $\sqrt{2}$ one of the most popular surds of all time.

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Advanced_Maths