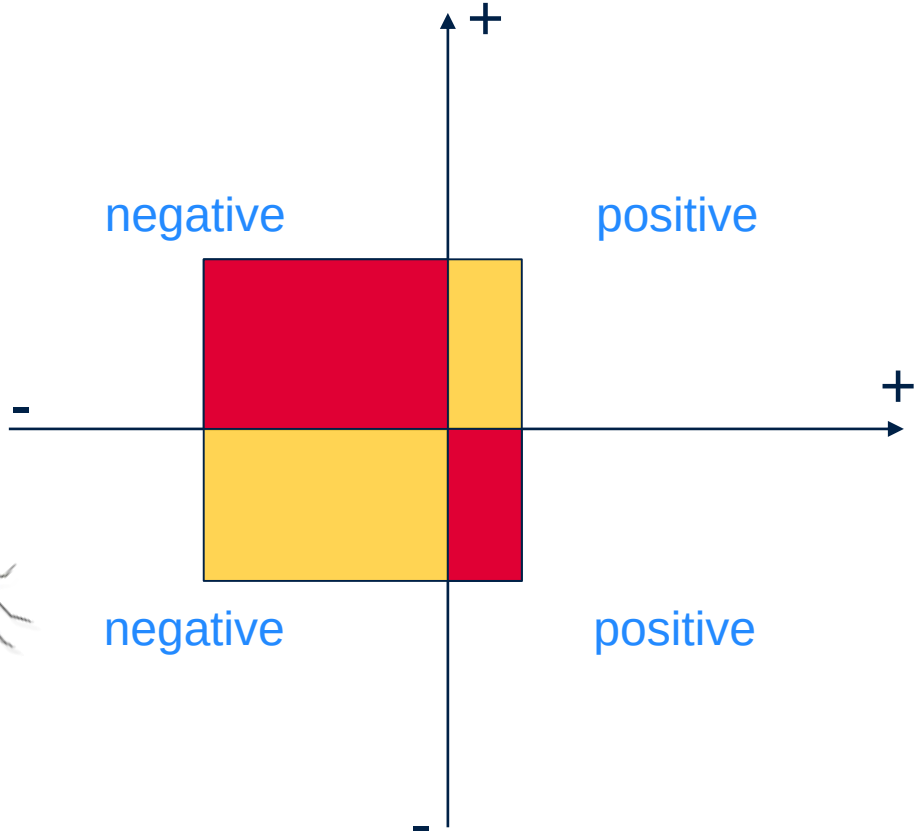
Advanced Mathematics Support Programme®

Did you know ?

?

We use a 2D co-ordinate plane to develop understanding regarding multiplication and also a *lot* of algebra.



It was developed by Rene Descarte, in 1637, when he lay in his sick bed - watching ants crawl across a tiled ceiling!

The Ancient Greeks did not have the notion of a co-ordinate plane but they used similar geometric methods to develop very sophisticated algebra over 2500 years ago!

How can the diagram help us understand what happens when you multiply with negative numbers?

Dealing with Negativity

Try these quick questions

What do you notice about your answers?

1. $4 \times (-7) \times 6$
2. $3 \times 9 \times (-6)$
3. $2 \times (-3) \times (-4)$
4. $2 \times (-2) \times (-2) \times (-5)$
5. $a \times 7 \times a$
6. $ab \times 3 \times 6b$
7. $(-4a) \times 7a \times (-6a)$

8. Use what you have noticed to fill in the gaps in the sentences below

positive | EVEN | negative | ODD

With an …….. number of negative numbers then value will be …….

With an …….. number of negative numbers then value will be …….

Dealing with Negativity

Solutions on the next slide….

Dealing with Negativity

Try these quick questions

What do you notice about your answers?

1. $4 \times (-7) \times 6 = -168$
2. $3 \times 9 \times (-6) = -162$
3. $2 \times (-3) \times (-4) = 24$
4. $2 \times (-2) \times (-2) \times (-5) = -40$
5. $(-a) \times (-7) \times a = 7a^2$
6. $ab \times 3 \times 6b = 18ab^2$
7. $(-4a) \times 7a \times (-6a) = 168a^3$

8. Use what you have noticed to fill in the gaps in the sentences below

With an EVEN number of negative numbers then value will be positive

With an ODD number of negative numbers then value will be negative

Unsure about any of these? Search Negative numbers

Expanding 1

1. Without doing the calculation, will the answer to this calculation be positive or negative ? Give a reason.

$2 \times (-3) \times (-4) \times 6 \times (-6) \times (-1) \times 7 \times (-2)$

2. 24×17 is the same as which of the following

$2 \times 3 \times 17 \times 2 \times 2$	$(20 + 4)(10 + 7)$
$(30 - 5)(20 - 2)$	$20(10 + 7) + 4(10 + 7)$

3. Expand $3(\sqrt{3} - 6)$

4. Expand and simplify $(x + 2)(x + 5)$

5. Expand and simplify $(x + 6)(x - 2)$

6. Expand and simplify $(\sqrt{2} + 3)(\sqrt{2} + 1)$

7. Expand and simplify $(x^2 + 2)(x^2 + 6)$

8. Expand and simplify $(x^2 + 3)(x^3 + 7)$

Expanding 1

Solutions on the next slide….

Expanding 1 Solutions

1. Without doing the calculation, will the answer to this calculation be positive or negative ? Give a reason.

$2 \times (-3) \times (-4) \times 6 \times (-6) \times (-1) \times 7 \times (-2)$

→ Negative because there are an odd number of negative numbers

2. 24×17 is the same as which of the following

$2 \times 3 \times 17 \times 2 \times 2$	$(20 + 4)(10 + 7)$
$(30 - 5)(20 - 2)$	$20(10 + 7) + 4(10 + 7)$

→

$2 \times 3 \times 17 \times 2 \times 2$	$(20 + 4)(10 + 7)$
$(30 - 5)(20 - 2)$	$20(10 + 7) + 4(10 + 7)$

3. Expand $3(\sqrt{3} - 6)$

→ $= 3\sqrt{3} - 18$

4. Expand and simplify $(x + 2)(x + 5)$

→ $= x^2 + 2x + 5x + 10$
$= x^2 + 7x + 10$

Unsure about any of these? Search Expanding single brackets

Expanding 1 Solutions

5. Expand and simplify $(x+6)(x-2)$ → $= x^2 + 6x - 2x - 12$
$= x^2 + 4x - 12$

6. Expand and simplify $(\sqrt{2}+3)(\sqrt{2}+1)$ → $= 2 + 3\sqrt{2} + \sqrt{2} + 3$
$= 4\sqrt{2} + 5$

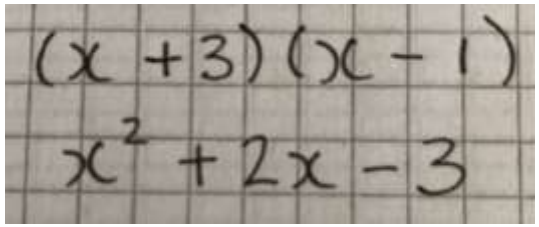
7. Expand and simplify $(x^2+2)(x^2+6)$ → $= x^4 + 2x^2 + 6x^2 + 12$
$= x^4 + 8x^2 + 12$

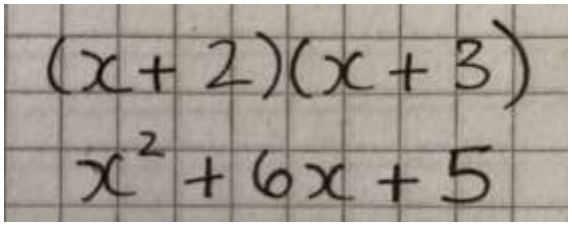
8. Expand and simplify $(x^2+3)(x^3+7)$ → $= x^5 + 3x^3 + 7x^2 + 21$

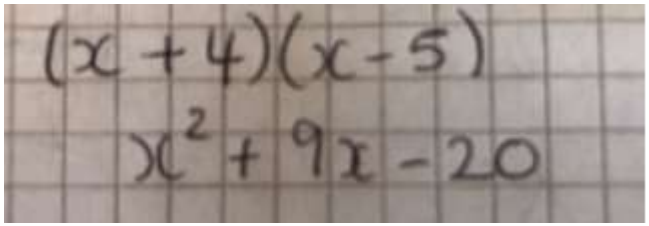
Unsure about any of these? Search Expanding Double Brackets

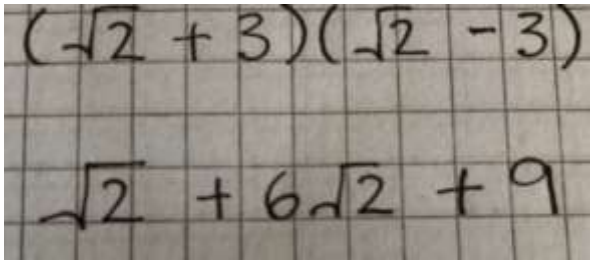
What's gone wrong?

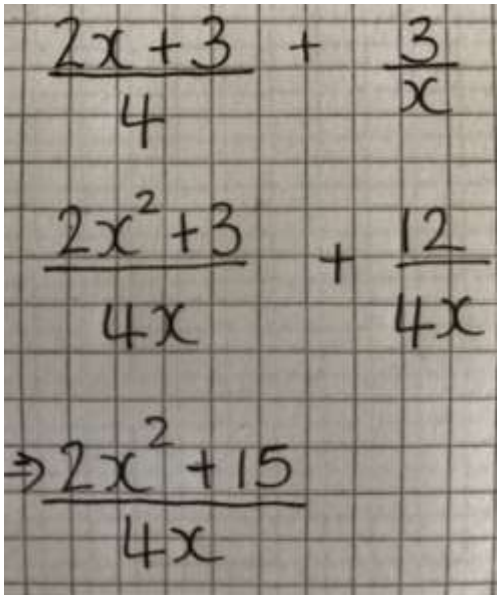
- Here is a student's work on expanding brackets.
- Take a look and decide if they have done the work correctly or not.
- If they have made a mistake can you say why ?
- What are the correct answers?

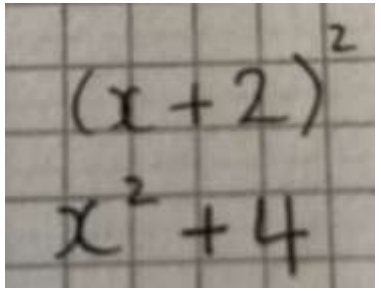












What's gone wrong?

Solutions on the next slide….

What's gone wrong? Solutions

- Here is a student's work on expanding brackets.
- Take a look and decide if they have done the work correctly or not.
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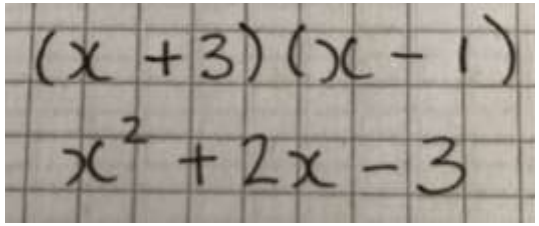
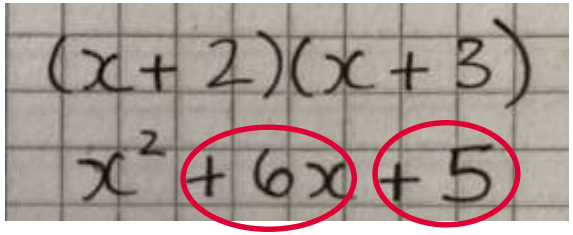
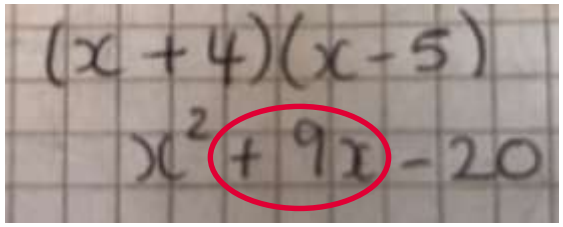


Correct



$+2x + 3x \quad +2 \times 3$

$= x^2 + 5x + 6$



$+4x - 5x$

$= x^2 - x - 20$

What's gone wrong? Solutions

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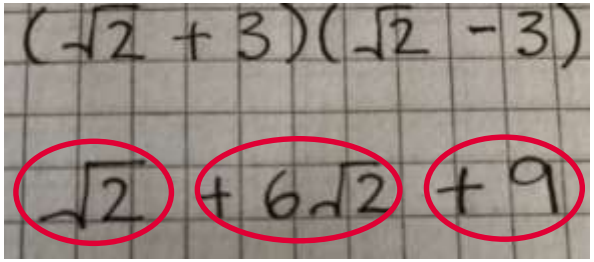
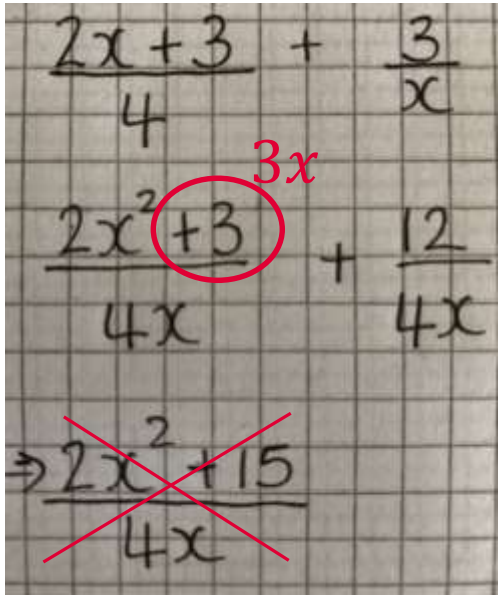


$\sqrt{2} \times \sqrt{2}$ $+3\sqrt{2} - 3\sqrt{2}$ $+3 \times -3$

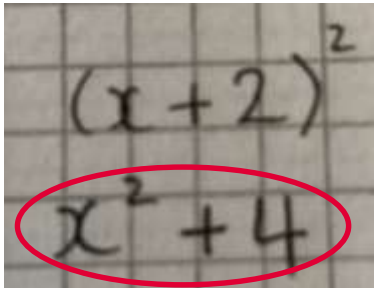
2 0 -9

$= 2 + 0 + (-9)$

$= -7$



$$= \frac{2x^2 + 3x + 12}{4x}$$



$= (x + 2)(x + 2)$

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

Expand and Simplify

- Expand the expressions on the left of the page and find the matching expression in the grid on the right of the page.
- When completed there should be four answers unmatched.
- Find the sum of these four expressions and simplify it

1. $(x+3)^2$
2. $(x+4)(x+3)$
3. $(x-4)^2-2$
4. $(x-3)(x+4)$
5. $(x+5)^2+3$
6. $x(x+4)+2(x+4)$
7. $(3-x)(3+x)$
8. $x(x-8)-(x-8)$

$x^2+6x-16$	x^2+6x+9	x^2+6x+8	x^2+9
$x^2+7x+12$	x^2-9x+8	$x^2-5x+12$	$x^2-8x+14$
$9-x^2$	$-x^2+6x+36$	$x^2+10x+28$	x^2+x-12

Expand and Simplify

Solutions on the next slide….

Expand and Simplify

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1. $(x+3)^2$
2. $(x+4)(x+3)$
3. $(x-4)^2-2$
4. $(x-3)(x+4)$
5. $(x+5)^2+3$
6. $x(x+4)+2(x+4)$
7. $(3-x)(3+x)$
8. $x(x-8)-(x-8)$

$x^2+6x-16$	1 x^2+6x+9	6 x^2+6x+8	x^2+9
2 $x^2+7x+12$	8 x^2-9x+8	$x^2-5x+12$	3 $x^2-8x+14$
7 $9-x^2$	$-x^2+6x+36$	5 $x^2+10x+28$	4 x^2+x-12

The four expressions left simplify to $2x^2+7x+41$

Quadratic Puzzles

- These are multiplication grids
- We can use these to expand quadratics such as $(x + 3)(x + 4)$

$(x + 3)$

$\times$	x	$+3$
x	x^2	$3x$
$+4$	$4x$	12

$(x + 4)$

$x^2 + 3x + 4x + 12$

There are 4 terms after expanding the brackets. That's why these expressions are called quadratics – as 'quad' means four.

Now we can simplify by collecting like terms to get this → $x^2 + 7x + 12$

- On the next page fill in the blanks in the multiplication grids
- What do you notice?

Quadratic Puzzles

$(x-4)$

$(x-4)$ ×	x	-4
x		
-4	$-4x$	$+16$

x^2 ☐ $+16$

$(2x+1)$

$(x+2)$ ×	$2x$	
x		
$+2$		$+2$

$2x^2$ ☐

$(3x-5)$

$(x+3)$ ×	$3x$	
x		
	$+9x$	

☐

$(2x+3)$

$(2x-4)$ ×		
$2x$		

☐

$(3x+4)$

$(2x-3)$ ×		

☐

$(5x-2)$

$(5x+2)$ ×		

☐

Quadratic Puzzles

Solutions on the next slide….

Quadratic Puzzles Solutions

$(x-4)$ $(x-4)$

×	x	-4
x	x^2	$-4x$
-4	$-4x$	$+16$

$x^2 - 8x + 16$

$(2x+1)$ $(x+2)$

×	$2x$	$+1$
x	$2x^2$	$+x$
$+2$	$+4x$	$+2$

$2x^2 + 5x + 3$

$(3x-5)$ $(x+3)$

×	$3x$	-5
x	$3x^2$	$-5x$
$+3$	$+9x$	-15

$3x^2 + 4x - 15$

$(2x+3)$ $(2x-4)$

×	$2x$	$+3$
$2x$	$4x^2$	$+6x$
-4	$-8x$	-12

$4x^2 - 2x - 12$

$(3x+4)$ $(2x-3)$

×	$3x$	$+4$
$2x$	$6x^2$	$+8x$
-3	$-9x$	-12

$6x^2 - x - 12$

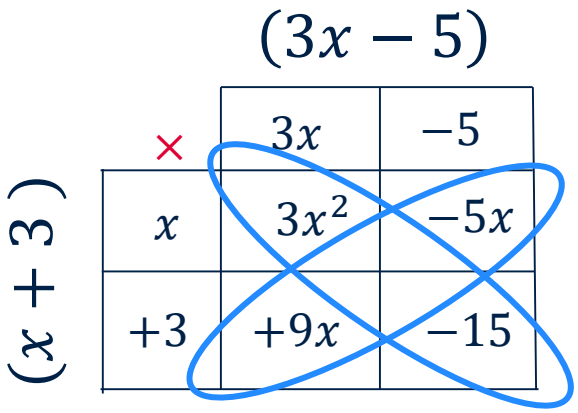
$(5x-2)$ $(5x+2)$

×	$5x$	-2
$5x$	$25x^2$	$-10x$
-2	$-10x$	-4

$25x^2 - 4$

Quadratic Puzzles Solutions

- What did you notice?



The sum of these terms make the middle term in the simplified expression

$3x^2 + 9x - 5x - 15$

$3x^2 + 4x - 15$

The products of the diagonals are identical expressions

$3x^2 \times -15 = -45x^2$

$9x \times -5x = -45x^2$

You might want to go back and check this with other quadratics yourself - this might be useful to know later on

Expanding 2

1. Expand and simplify $(2x + 3)(x - 2)$

2. Expand and simplify $3x(x + 3) + 4(x + 3)$

3. Expand and simplify $(x + 6)^2 + (x - 3)^2$

4. Expand and simplify $\left(2 - \sqrt{3}\right)^2$

5. Simplify $\frac{2}{(x+3)} + \frac{x-3}{x}$

6. Expand and simplify $(x^3 - 7)(x^3 + 7)$

7. Expand and simplify $(3x + 2)(4x^2 + 2x - 3)$

8. Simplify $\frac{2x-2}{(x+2)} - \frac{x-2}{3x}$

Expanding 2

Solutions on the next slide….

Expanding 2 Solutions

1. Expand and simplify $(2x + 3)(x - 2)$

$= 2x^2 + 3x - 4x - 6$
$= 2x^2 - x - 6$

2. Expand and simplify $3x(x + 3) + 4(x + 3)$

$= 3x^2 + 9x + 4x + 12$
$= 3x^2 + 13x + 12$

3. Expand and simplify $(x + 6)^2 + (x - 3)^2$

$= (x + 6)(x + 6) + (x - 3)(x - 3)$
$= x^2 + 12x + 36 + x^2 - 6x + 9$
$= 2x^2 + 6x + 45$

4. Expand and simplify $(2 - \sqrt{3})^2$

$= (2 - \sqrt{3})(2 - \sqrt{3})$
$= 4 - 2\sqrt{3} - 2\sqrt{3} + 3$
$= 7 - 4\sqrt{3}$

Expanding 2 Solutions

5. Simplify $\frac{2}{(x+3)}+\frac{x-3}{x}$

$$=\frac{2}{(x+3)}+\frac{x-3}{x} \rightarrow \frac{2x}{x(x+3)}+\frac{(x-3)(x+3)}{x(x+3)}$$

$$=\frac{2x}{x(x+3)}+\frac{x^2-9}{x(x+3)} \rightarrow \frac{x^2+2x-9}{x(x+3)}$$

6. Expand and simplify $(x^3-7)(x^3+7)$

$$=x^6-7x^3+7x^3-49$$
$$=x^6-49$$

7. Expand and simplify $(3x+2)(4x^2+2x-3)$

$$=3x(4x^2+2x-3)+2(4x^2+2x-3)$$
$$=12x^3+6x^2-9x+8x^2+4x-6$$
$$=12x^3+14x^2-5x-6$$

8. Simplify $\frac{2x-2}{(x+2)}-\frac{x-2}{3x}$

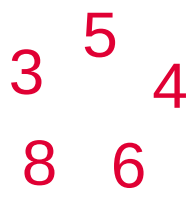
$$=\frac{3x(2x-2)}{3x(x+2)}-\frac{(x-2)(x+2)}{3x(x+2)} \rightarrow \frac{6x^2-6x}{3x(x+2)}-\frac{x^2-4}{3x(x+2)}$$

$$=\frac{5x^2-6x+4}{3x(x+2)}$$

Prove it!

Write some digits in a circle

e.g.



- The sum of the squares of the two-digit numbers read clockwise is

$$54^2 + 46^2 + 68^2 + 83^2 + 35^2 = 17770$$

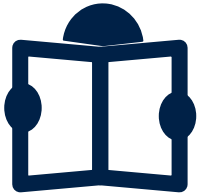
- The sum of the squares of the two-digit numbers read anticlockwise is

$$53^2 + 38^2 + 86^2 + 64^2 + 45^2 = 17770$$

Prove that the two sums will always be equal for any circle of digits

Hint Solution

Still want more?



Read more about how algebra was developed thousands of years ago and how visualisations were used even then!

Discover the history of negative numbers and how they were thought of as making dark of mathematics!

Watch this video to find how there are actually patterns in prime numbers and how simple algebra can show this – with brackets of course!

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