Advanced Mathematics Support Programme®

Have a think…

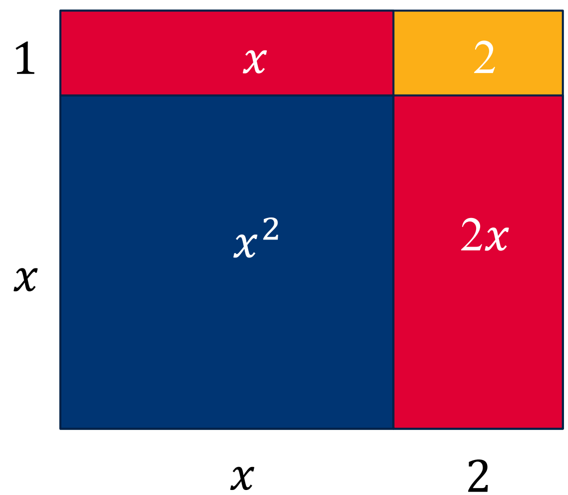
… about $(x+2)(x+1)$

Formal Method

$$(x+2)\,(x+1)$$
$$= x\,(x+1) + 2(x+1)$$
$$= x^2 + x + 2x + 2$$

$$= x^2 + 3x + 2$$

Geometrical Representation



$$= x^2 + 3x + 2$$

Grid Method

	x	$+2$
$+1$	x	2
x	x^2	$2x$

$$= x^2 + 3x + 2$$

- This expression expands to give 4 terms…which simplify to 3 terms.
- How many terms are in the unsimplified expansion of $(x+3)(x+4)(x+5)$?
- Be prepared to explain your thinking…

The story so far…..

1. Expand and simplify

$$(2x+3)(3x-5)$$

2. Write $(x+3)^2-4$ in the form ax^2+bx+c

3. Expand and simplify

$$(2a+2)(3x-4a+3)$$

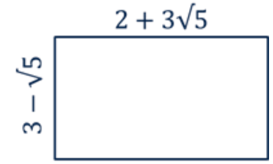
4. Expand and simplify

$$3x(x-3)(x+5)$$

5. Evaluate (no calc allowed)

$$\left(2+\frac{1}{3}\right)\left(2-\frac{1}{3}\right)$$

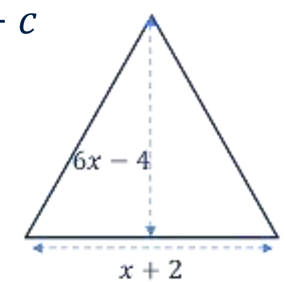
6. Find the area of this rectangle



7. Expand and simplify

$$(5-4x)(3x+6)+(5x-2)(3+4x)$$

8. Find the area of the triangle and write it in the form ax^2+bx+c



The story So Far……

Solutions on the next slide….

The story so far Solutions

1. Expand and simplify
$$(2x+3)(3x-5)$$

$$6x^2-10x+9x-15$$
$$6x^2-x-15$$

2. Write $(x+3)^2-4$ in the form ax^2+bx+c

$$x^2+6x+9-4$$
$$x^2+6x+5$$

3. Expand and simplify
$$(2a+2)(3x-4a+3)$$

$$6ax-8a^2+6a+6x-8a+6$$
$$6ax-8a^2-2a+6x+6$$

4. Expand and simplify
$$3x(x-3)(x+5)$$

$$3x(x^2+2x-15)$$
$$3x^3+6x^2-45x$$

Unsure about any of these? Search Expanding Brackets

The story so far Solutions

5. Evaluate (no calc allowed)

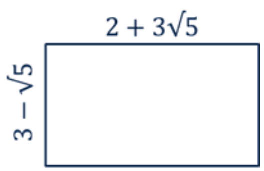
$$\left(2+\frac{1}{3}\right)\left(2-\frac{1}{3}\right)$$

Did you notice this is the difference of two squares?
$(a+b)(a-b)=a^2-b^2$

$$2^2-\left(\frac{1}{3}\right)^2$$

$$4-\frac{1}{9}=3\frac{8}{9}$$

6. Find the area of this rectangle



Area = $(2+3\sqrt{5})(3-\sqrt{5})$

$6-2\sqrt{5}+9\sqrt{5}-(3\sqrt{5}\times\sqrt{5})$

$7\sqrt{5}+(6-15)$

$7\sqrt{5}-9$

7. Expand and simplify

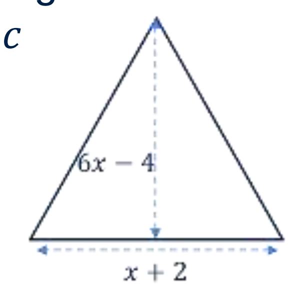
$$(5-4x)(3x+6)+(5x-2)(3+4x)$$

$15x+30-12x^2-24x+15x+20x^2-6-8x$

$20x^2-12x^2+15x+15x-24x-8x+30-6$

$8x^2-2x+24$

8. Find the area of the triangle and write it in the from ax^2+bx+c



Area = $\frac{1}{2}$ x height x base

$\frac{1}{2}(6x-4)(x+2)$

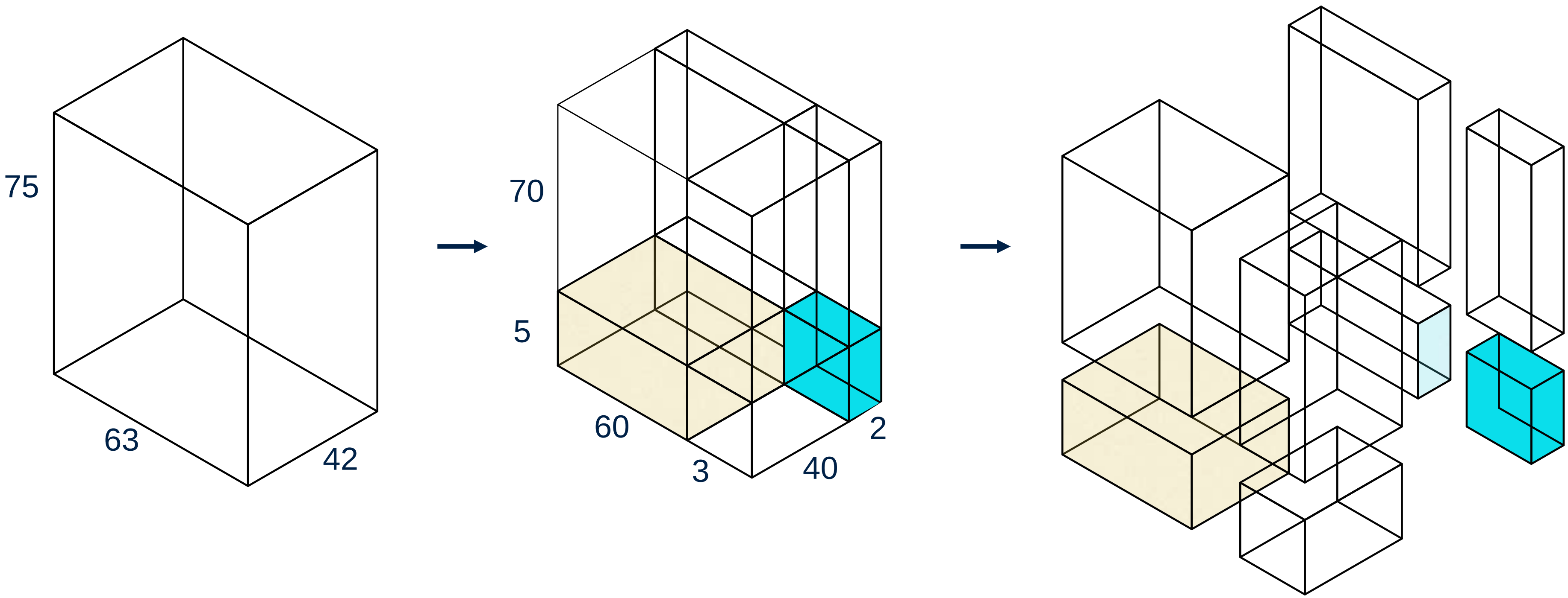
$(3x-2)(x+2)$

$3x^2+4x-4$

Unsure about any of these? Search Expanding Brackets

Have a look…

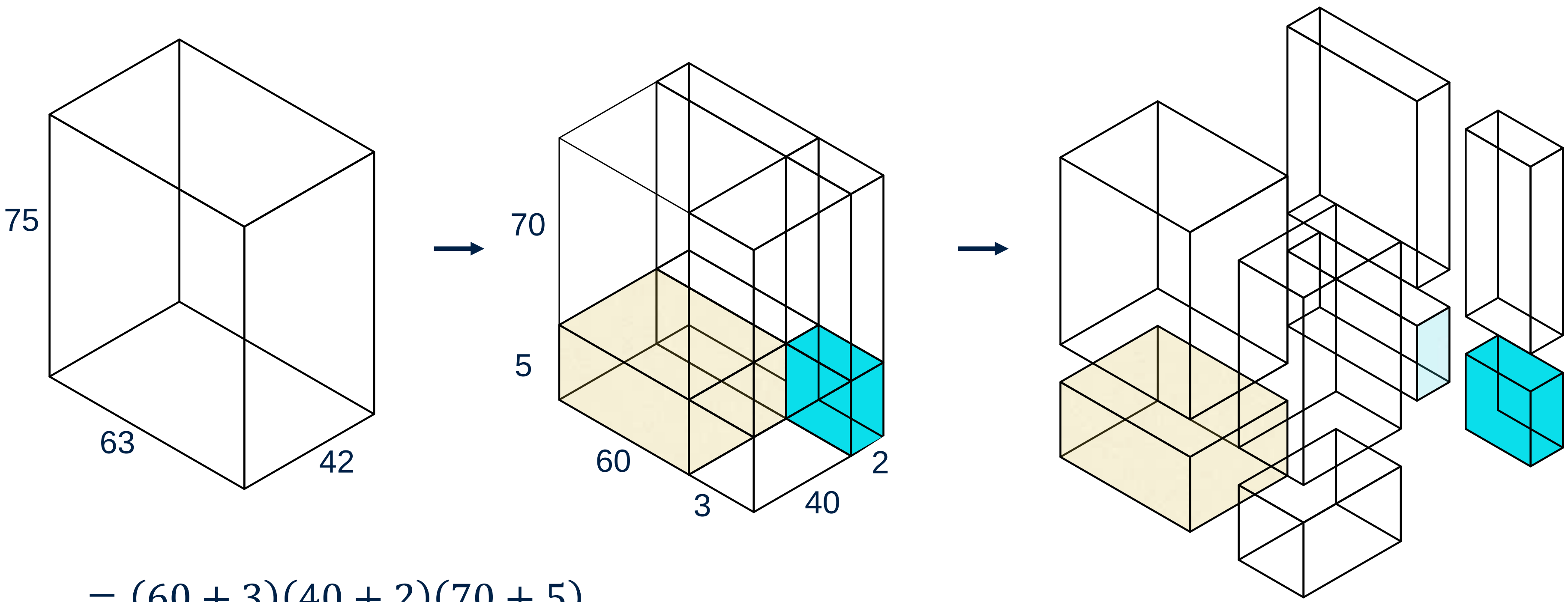
These diagrams represent a method for calculating 63 x 42 x 75



- Can you see what is going on?
- What are the values of the coloured sections?

Have a look…

These diagrams represent a method for calculating 63 x 42 x 75



$$
\begin{aligned}
&= (60+3)(40+2)(70+5) \\
&= (60\times 40\times 70) + (60\times 40\times 5) + (60\times 2\times 70) + (60\times 2\times 5) \\
&\quad + (3\times 40\times 70) + (3\times 40\times 5) + (3\times 2\times 70) + (3\times 2\times 5) \\
&= 198\,450
\end{aligned}
$$

This helps us answer the question on the first slide.

How many terms will there be when we expand $(x+3)(x+4)(x+5)$? 8 terms.

Think again

How might you go about
expanding the following?

$$(x+2)(x+1)(x+2)$$

- Is there a way you could use the working on the previous slide to help?
- How would the geometric diagram have to change?
- Can you use a grid method to speed things up?

Think again

How might you go about expanding the following?

$$(x + 2)(x + 1)(x + 2)$$

- Is there a way you could use the working on the previous slide to help?

The $(x + 2)(x + 1)(x + 2)$ expansion is the same as the 'splitting' of $63 \text{ x } 42 \text{ x } 75$ into $(60 + 3)(40 + 2)(70 + 5)$ in the initial task. Therefore we can just substitute into the long line of working:

$=(x \times x \times x) + (x \times x \times 2) + (x \times 1 \times x) + (x \times 1 \times 2) + (2 \times x \times x) + (2 \times x \times 2) + (2 \times 1 \times x) + (2 \times 1 \times 2)$

$= x^3 + 2x^2 + x^2 + 2x + 2x^2 + 4x + 2x + 4$

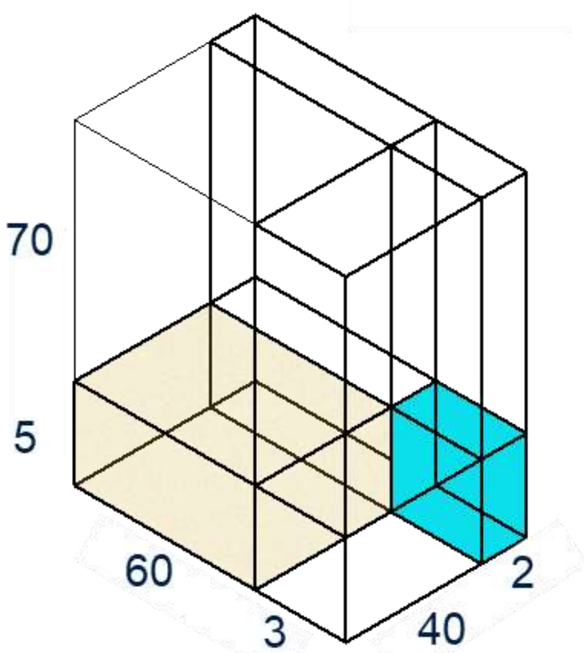
$= x^3 + 5x^2 + 8x + 4$

Think again

How might you go about expanding the following?

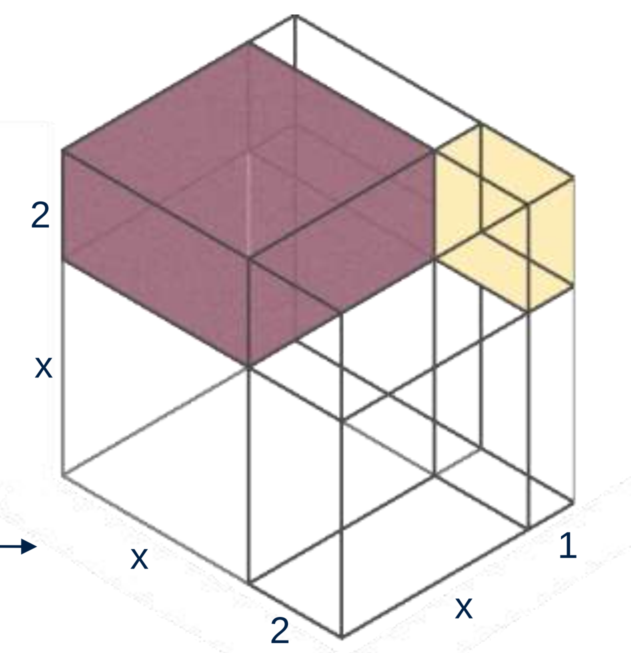
$$(x+2)(x+1)(x+2)$$

- How would the geometric diagram have to change?



Notice there is an x in each of the brackets in the question above.

You should be able to see this represented by a cube in the bottom left corner of the diagram.



Think again

How might you go about expanding the following?

$$(x+2)(x+1)(x+2)$$

- Can you use a grid method to speed things up?

First expand two of the brackets as shown

$$(x+2)\underbrace{(x+1)(x+2)}$$

	x	1
x	x^2	x
2	$2x$	2

$$(x+2)(x^2+3x+2)$$

Now use the expression obtained from the first step to set up another grid that is 3 x 2

$$\underbrace{(x+2)}(x^2+3x+2)$$

	x	2
x^2	x^3	$2x^2$
$3x$	$3x^2$	$6x$
2	$2x$	4

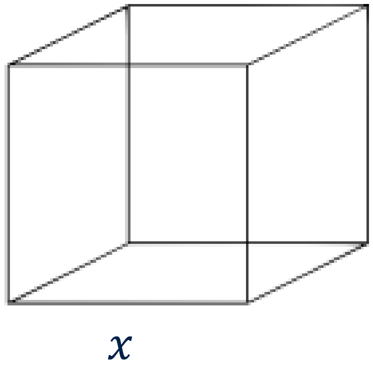
$$x^3+2x^2+3x^2+6x+2x+4$$

Notice how the grid method remains in 2D by ‘stepping’.

This is the equivalent of calculating the volume of a prism as cross-sectional area *x* perpendicular height.

$$= x^3+5x^2+8x+4$$

Getting bigger



Here is a cube with side lengths of x cm

The cube is going to have its lengths increased in one of three ways

Method A

Each side is increased by 2 units

Method B

One side is increased by 3 units, one side is increased by 2 units, and one side is increased by 1 unit

Method C

One side is increased by 5 units, one side is increased by 2 units, and one side is decreased by 1 unit

Can you prove which of the new solids will have the largest volume?

Getting bigger

Solutions on the next slide….

Getting bigger

Method A	Method B	Method C
Each side is increased by 2 units	One side is increased by 3 units, one side is increased by 2 units, and one side is increased by 1 unit	One side is increased by 5 units, one side is increased by 2 units, and one side is decreased by 1 unit

A. $(x+2)(x+2)(x+2)$

$= (x+2)(x^2+4x+4)$
$= x^3+4x^2+4x+2x^2+8x+8$
$= x^3+6x^2+12x+8$

B. $(x+3)(x+2)(x+1)$

$= (x+3)(x^2+3x+2)$
$= x^3+3x^2+2x+3x^2+9x+6$
$= x^3+6x^2+11x+8$

C. $(x+5)(x-1)(x+2)$

$= (x+5)(x^2+x-2)$
$= x^3+x^2-2x+5x^2+5x-10$
$= x^3+6x^2+3x-10$

Because x is a side length we know that x is positive.
Therefore A is the greatest as $12x+8$ is larger than $11x+8$ and $3x-10$

Will this method always work?
Can you find a set of changes where it is not possible to tell?

Expanding Cubics and beyond

Previously we saw how we could use a grid for expanding brackets such as $(1+x)^2$

	$\mathbf{1}$	$+\boldsymbol{x}$
$\mathbf{1}$	1	$+x$
$+\boldsymbol{x}$	$+x$	$+x^2$

So $(1+x)^2 = 1 + 2x + 1x^2$

	$\mathbf{1}$	$+\mathbf{2}\boldsymbol{x}$	$+\boldsymbol{x}^{\mathbf{2}}$
$\mathbf{1}$	1	$+2x$	$+x^2$
$+\boldsymbol{x}$	$+x$	$+2x$	$+x^3$

So $(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$

Can you use a similar approach to expand

- $(1+x)^4$
- $(1+x)^5$
- $(1+x)^6$

Expanding Cubics and beyond

Solutions on the next slide….

Expanding Cubics Solutions

	$\mathbf{1}$	$+\mathbf{3x}$	$+\mathbf{3x^2}$	$+\mathbf{x^3}$
$\mathbf{1}$	1	$+3x$	$+3x^2$	$+x^3$
$+\mathbf{x}$	$+x$	$+3x^2$	$+3x^3$	$+x^4$

So $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + 1x^4$

	$\mathbf{1}$	$+\mathbf{4x}$	$+\mathbf{6x^2}$	$+\mathbf{4x^3}$	$+\mathbf{x^4}$
$\mathbf{1}$	1	$+4x$	$+6x^2$	$+4x^3$	$+x^4$
$+\mathbf{x}$	$+x$	$+4x^2$	$+6x^3$	$+4x^4$	$+x^5$

So $(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + 1x^5$

	$\mathbf{1}$	$+\mathbf{5x}$	$+\mathbf{10x^2}$	$+\mathbf{10x^3}$	$+\mathbf{5x^4}$	$+\mathbf{x^5}$
$\mathbf{1}$	1	$+5x$	$+10x^2$	$+10x^3$	$+5x^4$	$+x^5$
$+\mathbf{x}$	x	$+5x^2$	$+10x^3$	$+10x^4$	$+5x^5$	$+x^6$

So $(1+x)^6 = 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + 1x^6$

Do you notice anything about the coefficients in the expansions?

Pascal's Triangle

Have you seen this triangle before?

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

- Look back at the coefficients you found
- Can you see any connection to the numbers in the triangle?

Pascal's Triangle Expansions

If we put the coefficients into a triangular pattern …

Row 0: $(1+x)^0 = 1$ → 1

Row 1: $(1+x)^1 = 1 + x$ → 1 1

1 2 1

Row 3: $(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$ → 1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Row 6: $(1+x)^6 = 1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + 1x^6$

- The coefficients are the numbers in a row of Pascal's triangle.
- The coefficients in the expansion of $(1+x)^4$ are the numbers in row 4 of Pascal's triangle

Pascal's Triangle Expansions

Use the triangle to help you

Complete these expansions

- $(1+x)^7$
- $(1+x)^8$

						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		6		4		1		
	1		5		10		10		5		1	
1		6		15		20		15		6		1

For the link to the solutions to the above questions - and to find out to extend these ideas further into AS/A level Mathematics – see the next slide.

Pascal’s Triangle Expansions

Follow the link to the solutions

Summary and Review

1. Expand and simplify
$$\left(\frac{1}{3}x+\frac{1}{9}\right)\left(3x-\frac{2}{3}\right)$$

2. Expand and simplify
$$(x+1)(x+2)(x+3)$$

3. Expand and simplify
$$(x-3)(x+2)^2$$

4. Expand and simplify
$$(2-\sqrt{3})(1+\sqrt{3})(1-\sqrt{3})$$

5. Find the volume of a cube with side length $x-4$

6. Expand and simplify
$$(x^2-2)(x^2+2)(x+1)$$

7. Write $\left(\sqrt{y}+\sqrt{8y}\right)^2$ in the form $a+b\sqrt{2}$.

   Given that $\left(\sqrt{y}+\sqrt{8y}\right)^2 = 54+b\sqrt{2}$.
   Find values for y and b.

8. Simplify $\frac{(x-1)(x+2)}{(x+3)} - \frac{4}{2x+1}$

Summary and Review

Solutions on the next slide….

Summary and review Solutions

1. Expand and simplify

$$\left(\frac{1}{3}x+\frac{1}{9}\right)\left(3x-\frac{2}{3}\right)$$

$$\left(\frac{1}{3}x+\frac{1}{9}\right)\left(3x-\frac{2}{3}\right)$$
$$x^2-\frac{2}{9}x+\frac{3}{9}x-\frac{2}{27}$$
$$x^2+\frac{1}{9}x-\frac{2}{27}$$

2. Expand and simplify

$$(x+1)(x+2)(x+3)$$

$$(x^2+3x+2)(x+3)$$
$$x^3+3x^2+2x+3x^2+9x+6$$
$$x^3+6x^2+11x+6$$

3. Expand and simplify

$$(x-3)(x+2)^2$$

$$(x-3)(x^2+4x+4)$$
$$x^3-3x^2+4x^2-12x+4x-12$$
$$x^3+x^2-8x-12$$

4. Expand and simplify

$$(2-\sqrt{3})(1+\sqrt{3})(1-\sqrt{3})$$

* Did you notice this is a difference of two squares?

$$(2-\sqrt{3})\left(1^2-(\sqrt{3})^2\right)$$
$$(2-\sqrt{3})(1-3)$$
$$-2(2-\sqrt{3})$$
$$2\sqrt{3}-4$$

Summary and review Solutions

5. Find the volume of a cube with side length $x-4$

$$(x-4)(x-4)(x-4)$$
$$(x-4)(x^2-8x+16)$$
$$x^3-8x^2+16x-4x^2+32x-64$$
$$x^3-12x^2+48x-64$$

Solution

6. Expand and simplify
$$(x^2-2)(x^2+2)(x+1)$$

* Did you notice this is a difference of two squares?

$$\left((x^2)^2-(2)^2\right)(x+1)$$
$$(x^4-4)(x+1)$$
$$x^5+x^4-4x-4$$

7. Write $\left(\sqrt{y}+\sqrt{8y}\right)^2$ in the form $a+b\sqrt{2}$.

Given that $\left(\sqrt{y}+\sqrt{8y}\right)^2 = 54+b\sqrt{2}$.
Find values for y and b.

$$\left(\sqrt{y}+\sqrt{8y}\right)\left(\sqrt{y}+\sqrt{8y}\right)$$
$$(\sqrt{y})^2+\left(\sqrt{8}\,x\left(\sqrt{y^2}\right)\right)+\left(\sqrt{8}\,x\left(\sqrt{y^2}\right)\right)+\left(\sqrt{8y}\right)^2$$
$$y+2y\sqrt{8}+8y=9y+4y\sqrt{2}$$

As $9y+4y\sqrt{2}=54+b\sqrt{2}$
9y = 54 so y = 6
4y=b so b = 4 x 6 = 24

8. Simplify $\frac{(x-1)(x+2)}{(x+3)}-\frac{4}{2x+1}$

$$\frac{(2x+1)(x-1)(x+2)-4(x+3)}{(x+3)(2x+1)}$$

$$\frac{(2x^3+2x^2-4x+x^2+x-2)-(4x+12)}{(x+3)(2x+1)}$$
$$\frac{2x^3+3x^2-7x-14}{(x+3)(2x+1)}$$

Still want more?

Read more about Pascal's triangle, interact with it and find out more about it's heritage and who really discovered it first!

Discover more expansions linking to geometrical representations. You'll find a hint and a potential solution from other students to help you too.

Watch this video and encounter the almost endless amount of number patterns contained within Pascal's triangle.

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Advanced_Maths