Advanced Mathematics Support Programme®

Did you know?

How would you work out $32 \div 8$?

- Did you use the fact that you know $8 \times 4 = 32$?

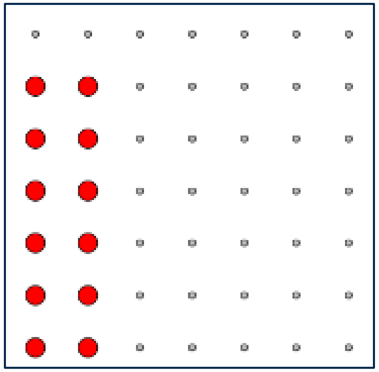
Often we use multiplication to help us do division as it is more straightforward.

- The same is true for factorising and expanding.
- It can often be easier to expand than to factorise
- So use expanding to help you factorise

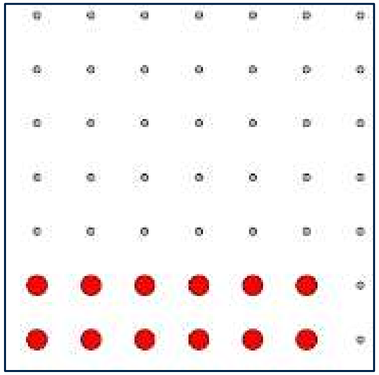
Factors and Rectangles

I have 12 red counters and a large sheet of dotted paper. How many different rectangular arrays can I make using all 12 counters?

A



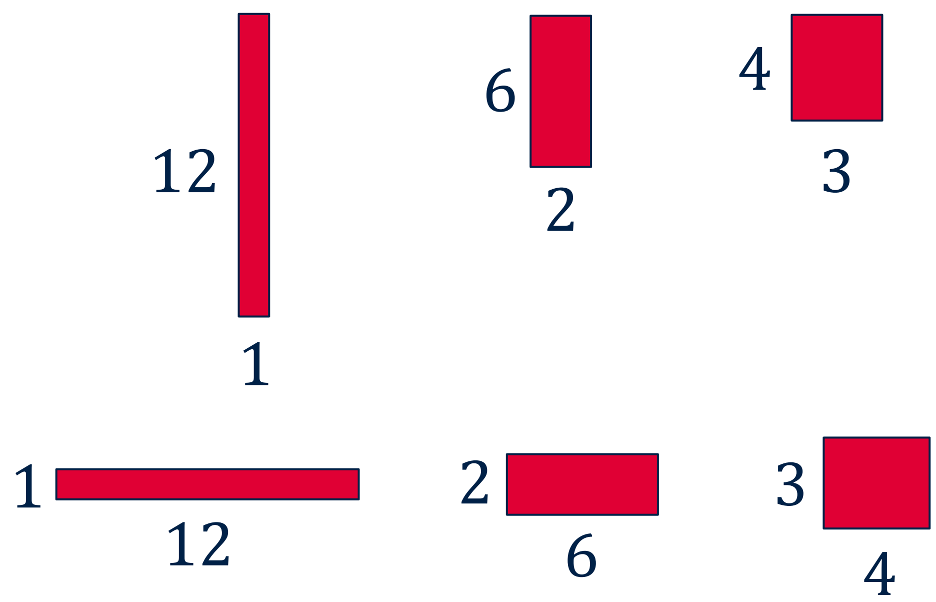
B



- An array is an arrangement of objects in rows and columns
- For this activity we will count A and B as different arrays as they have different orientations

Factors and Rectangles

This problem is equivalent to finding the number of rectangles with area 12 that have integer length sides, and counting 2 by 6 as different to 6 by 2



There are six arrays for 12 counters.

Factoring and Rectangles

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How many different arrays are there for:

- 7 counters?
- 15 counters?
- 25 counters?
- A prime number of counters?
- What is special about numbers with an odd number of arrays?

Factoring and Rectangles Solutions

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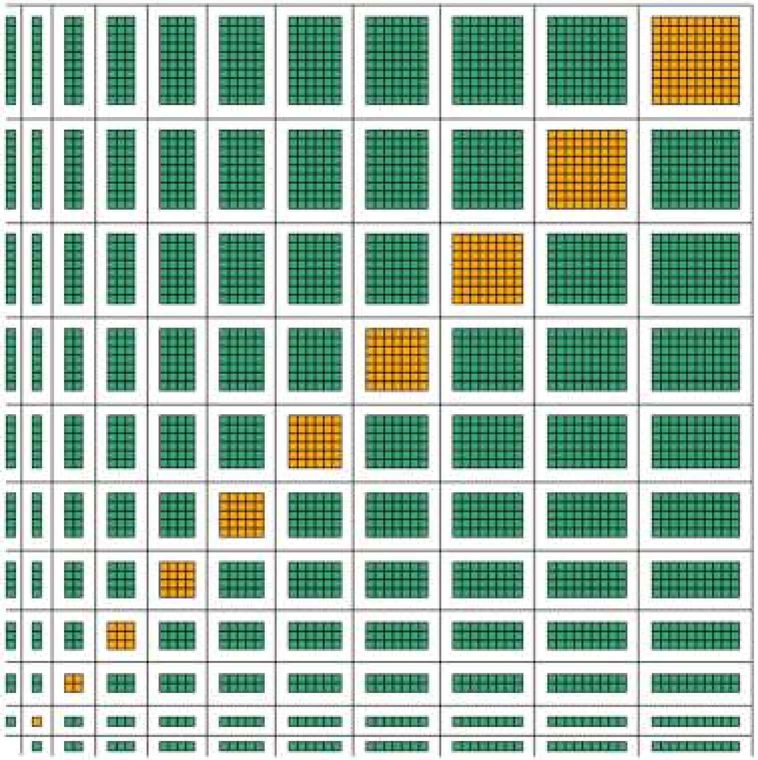
No of Counters	No of Arrays	
7	2	$(1 \times 7), (7 \times 1)$
15	4	$(1 \times 15), (3 \times 5), (5 \times 3), (15 \times 1)$
25	3	$(1 \times 25), (5 \times 5), (25 \times 1)$
Prime	2	$(1 \times p), (p \times 1)$
1	1	(1×1)
Square number	Odd number	

Remember the Visual Multiplication Square from the Expanding session? How does that help?



Factorising 1

Fully factorise the following:

1. $5x - 30$
2. $9x + 6$
3. $x^2 + 6x$
4. $6y^3 - 12y$
5. $7a^2b + 21ab - 14a$
6. $12x^2 + 12xy + 12y^2$
7. $3t(t - 1) + 7(t - 1)$
8. $2x(x^2 + 3) - 5(x^2 + 3)$

Factorising 1

Solutions on the next slide….

Factorising 1 Solutions

Fully factorise the following:

1. $5x - 30$ → $= 5(x - 6)$

2. $9x + 6$ → $= 3(3x + 2)$

3. $x^2 + 6x$ → $= x(x + 6)$

4. $6y^3 - 12y$ → $= 6y(y^2 - 2)$

Factorising 1 Solutions

Fully factorise the following:

5. $7a^2b + 21ab - 14a$ → $= 7a(ab + 3b - 2)$

6. $12x^2 + 12xy + 12y^2$ → $= 12(x^2 + xy + y^2)$

7. $3t(t-1) + 7(t-1)$ →

The common factor to take out is $(t-1)$

$3t(t-1) + 7(t-1)$

$= (t-1)(3t+7)$

8. $2x(x^2+3) - 5(x^2+3)$ →

The common factor to take out is (x^2+3)

$2x(x^2+3) - 5(x^2+3)$

$= (x^2+3)(2x-5)$

Unsure about any of these? Search Factorising single brackets. Next try Skills check 2….

Factorising 2

Fully factorise the following:

1. $7x + 28$
2. $14 - 21x$
3. $y^2 - 8y$
4. $3t^4 + 9t^2$
5. $3x^3y - 12xy^2 + 6xy$
6. $8a^3b + 6y^2b - 10b$
7. $6x(x+3) + 5(x+3)$
8. $7y(3-2y) - 2(3-2y)$

You can do this for fun - or move on if you correctly completed Skills check 1.

Factorising 2

Solutions on the next slide….

Factorising 2 Solutions

Fully factorise the following:

1. $7x + 28$ → $= 7(x + 4)$

2. $14 - 21x$ → $= 7(2 - 3x)$

3. $y^2 - 8y$ → $= y(y - 8)$

4. $3t^4 + 9t^2$ → $= 3t^2(t^2 + 3)$

Factorising 2 Solutions

Fully factorise the following:

5. $3x^3y - 12xy^2 + 6xy$ → $= 3xy(x^2 - 4y + 2)$

6. $8a^3b + 6y^2b - 10b$ → $= 2b(4a^3 + 3y^2 - 5)$

7. $6x(x+3) + 5(x+3)$ →

The common factor to take out is $(x+3)$

$6x(x+3) + 5(x+3)$

$= (x+3)(6x+5)$

8. $7y(3-2y) - 2(3-2y)$ →

The common factor to take out is $(3-2y)$

$7y(3-2y) - 2(3-2y)$

$= (3-2y)(7y-2)$

Enough Information

You are told that

$$ab = 245$$
$$bc = 635$$
$$a + c = 88$$

What is the value of b?

Hints available on the next slide

Enough Information Hints

You are told that

$$ab = 245$$
$$bc = 635$$
$$a + c = 88$$

What is the value of b?

- Try adding the first two expressions together
- Now factorise
- Have another look at the question

Enough Information Solution

You are told that

$$ab = 245$$

$$bc = 635$$

$$a + c = 88$$

What is the value of b?

$$ab + bc = 245 + 635$$

$$b(a + c) = 880$$

$$b(88) = 880$$

$$b = 10$$

Square Root

By considering prime factors, and without a calculator, find the square root of 27×147

Hints available on the next slide

Square Root Hints

By considering prime factors, and without a calculator, find the square root of 27×147

- Draw prime factor trees for 27 and 147 separately
- Write down 27 x 147 expressed as a product of their prime factors
- Simplify the expression
- Have another look at the question

Square Root Solution

By considering prime factors, and without a calculator, find the square root of 27×147

$27 = 3^3$

$147 = 3 \times 7^2$

Therefore $27 \times 147 = 3^3 \times 3 \times 7^2 = 3^4 \times 7^2$

$$\sqrt{27 \times 147} = \sqrt{3^4 \times 7^2} = \sqrt{3^4} \times \sqrt{7^2} = 3^2 \times 7 = 63$$

The Root Cause

Simplify $\sqrt{2y^2(x+3)^2+7(x+3)^2y^2}$

Hint available on the next slide

The Root Cause Hint

Simplify $\sqrt{2y^2(x+3)^2 + 7(x+3)^2y^2}$

- Factorise first (Q7 and Q8 from Factorising 1 ☑ will help)
- Have another look at the question

The Root Cause Solution

Simplify $\sqrt{2y^2(x+3)^2+7(x+3)^2y^2}$

The common factor is $(x+3)^2$

$$\sqrt{2y^2(x+3)^2+7(x+3)^2y^2}$$

$$=\sqrt{(x+3)^2(2y^2+7y^2)}$$

$$=\sqrt{9y^2(x+3)^2}$$

$$=3y(x+3)$$

Power Puzzle

Simplify

$$\frac{4x^{2.5} - 6\sqrt{x}}{2x^2 - 3}$$

Hints available on the next slide

Power Puzzle Hint

Simplify

$$\frac{4x^{2.5} - 6\sqrt{x}}{2x^2 - 3}$$

- Rewrite $\sqrt{x}$ as a power of x
- What is 2.5 as a fraction?
- Factorise the numerator
- Have another look at the question

Power Puzzle Solution

Simplify

$$\frac{4x^{2.5} - 6\sqrt{x}}{2x^2 - 3}$$

$$= \frac{4x^{2.5} - 6x^{0.5}}{2x^2 - 3}$$

$$= \frac{2x^{0.5}(2x^2 - 3)}{2x^2 - 3} = 2x^{0.5} = 2\sqrt{x}$$

Factor Problem

Pick 3 different integers from 1 to 10

Place your numbers in the boxes in as many different ways as possible (i.e 6 ways)

$$\square(\square x + \square)$$

- Write down all the expressions
- Multiply them all out
- Add up all your results and simplify
- Now factorise that answer

Try again with a different set of 3 numbers

What do you notice? Can you prove it?

Factor Problem

Follow the link for hints and solutions

Still want more?

Read about the amazing properties of prime numbers. Generate large primes for yourself and find out how you can make money from solving prime number problems.

Discover how you can use place value and factorising to explore number tricks by attempting this nrich problem.

Watch this video by James Grime. See if you can work out why the trick works.

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