Advanced Mathematics
Support Programme®

Did you know?

?

Substitute $x = 9$ into the following two expressions

$$x^2 + 3x + 2$$

and

$$(x+2)(x+1)$$

What do you notice?

Did you know?

Substitute $x = 9$ into the following two expressions

$$x^2 + 3x + 2$$

$$(9)^2 + 3(9) + 2 = 81 + 27 + 2 = 110$$

and

$$(x + 2)(x + 1)$$

$$(9 + 2)(9 + 1) = 11 \times 10 = 110$$

Both give the same answer as the expressions are equivalent

One of the expressions was a lot easier to evaluate! Why?

Which is best?

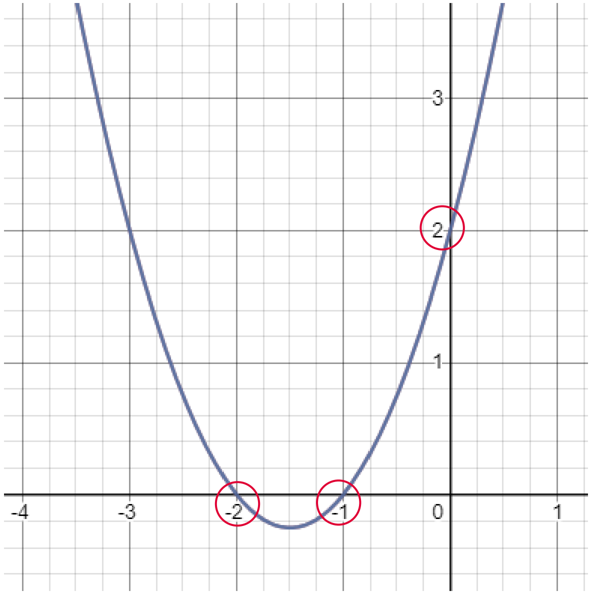
$$x^2 + 3x + 2 \ \textit{or} \ (x+2)(x+1)$$

expanded form

factorised form

$y = x^2 + 3x + 2$

$y = (x+2)(x+1)$



Factorising is a key skill for both sketching graphs and solving equations, both of which will be covered later.

Sometimes it is more helpful to factorise an expression, other times better to be expand it, depending on the context.

Further Factorising 1

Factorise the following fully:

1. $x^2 + 5x - 6$

2. $x^2 + 13x - 30$

3. $y^2 - 13y + 30$

4. $t^2 + 2t - 15$

5. $k^2 - 2k - 24$

6. $p^2 - 10p + 21$

7. $x^2 - 16x$

8. $3x(2x - 1) + 4(1 - 2x)$

Further Factorising 1

Solutions on the next slide….

Further Factorising 1 Solutions

1. $x^2 - 5x + 6$ $\rightarrow$ $= (x + 6)(x - 1)$

2. $x^2 + 13x - 30$ $\rightarrow$ $= (x + 15)(x - 2)$

3. $y^2 - 13y + 30$ $\rightarrow$ $= (y - 10)(y - 3)$

4. $t^2 + 2t - 15$ $\rightarrow$ $= (t + 5)(t - 3)$

Unsure about any of these? Search Factorising quadratics. Next try Skills check 2….

Further Factorising 1 Solutions

5. $k^2 - 2k - 24$ → $= (k-6)(k+4)$

6. $p^2 - 10p + 21$ → $= (p-7)(p-3)$

7. $x^2 - 16x$ → $= x(x-16)$

8. $3x(2x-1) + 4(1-2x)$ →

Can you see $-(2x-1)$ is the same as $(1-2x)$

Take -1 out as a factor

$= 3x(2x-1) - 4(2x-1)$

The common factor to take out is $(2x-1)$

$= (2x-1)(3x-4)$

Unsure about any of these? Search Factorising quadratics. Next try Skills check 2….

Further Factorising 2

Factorise the following fully:

1. $x^2 + 6x - 7$
2. $y^2 + y - 12$
3. $y^2 - 11y + 28$
4. $t^2 - 7t - 18$
5. $k^2 + 9k + 20$
6. $x^2 + x - 56$
7. $p^2 - 25p$
8. $x^2(3x - 4) + (4 - 3x)$

You can do this for fun - or move on if you correctly completed Skills check 1.

Further Factorising 2

Solutions on the next slide….

Further Factorising 2 Solutions

1. $x^2 + 6x - 7 \rightarrow = (x+7)(x-1)$

2. $y^2 + y - 12 \rightarrow = (y+4)(y-3)$

3. $y^2 - 11y + 28 \rightarrow = (y-7)(y-4)$

4. $t^2 - 7t - 18 \rightarrow = (t-9)(t+2)$

Further Factorising 2 Solutions

5. $k^2 + 9k + 20$ $\rightarrow$ $= (k+5)(k+4)$

6. $x^2 + x - 56$ $\rightarrow$ $= (x+8)(x-7)$

7. $p^2 - 25p$ $\rightarrow$ $= p(p-25)$

Did you notice? $-(3x-4)$ is the same as $(4-3x)$

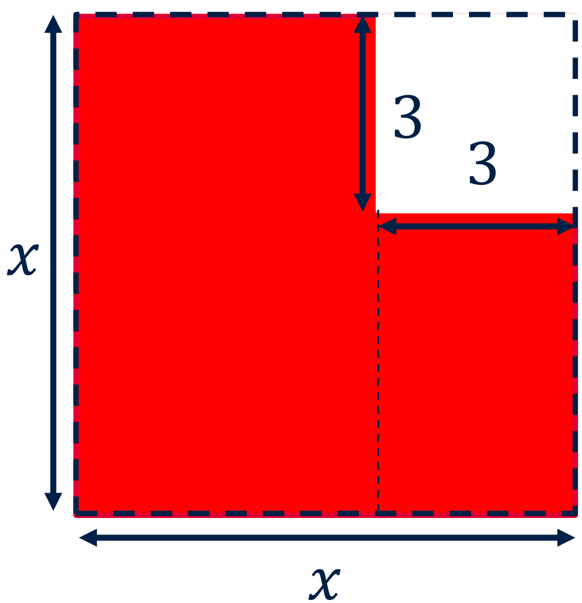
8. $x^2(3x-4) + (4-3x)$ $\rightarrow$ $= x^2(3x-4) - (3x-4)$

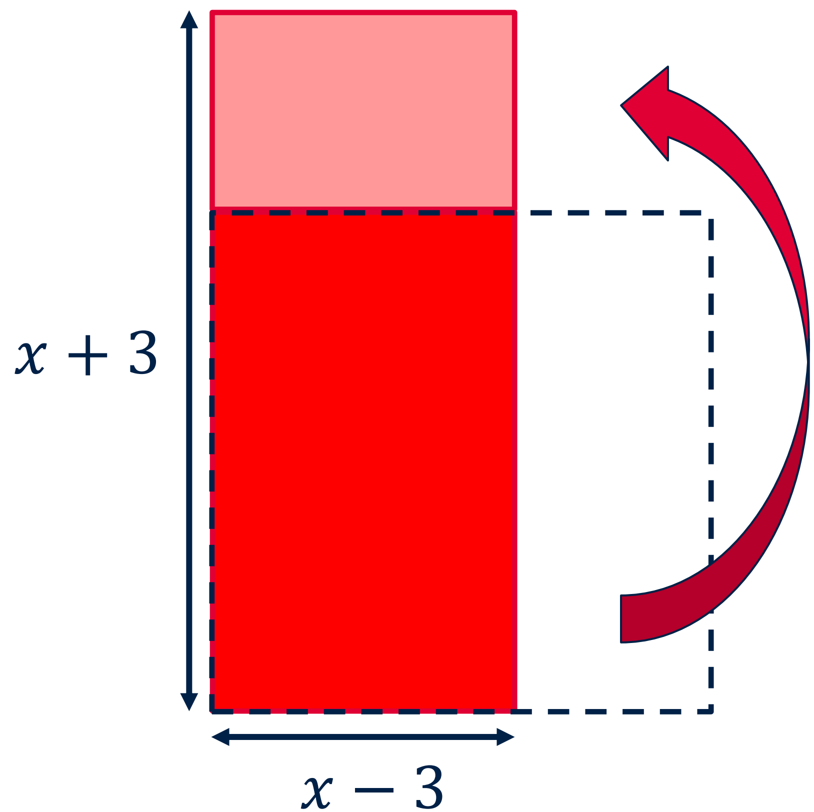
The common factor to take out is $(3x-4)$

$= (3x-4)(x^2-1)$

Difference of two squares

A special case for factorising is the difference of two squares. Expressions such as $x^2 - 3^2$, where the coefficient of x is zero.





$$x^2 - 3^2 \quad = \quad (x-3)(x+3)$$

Difference of two squares

Try factorising these expressions using the difference of two squares

1. $x^2 - 6^2$
2. $y^2 - 144$
3. $x^2 - y^2$
4. $4t^2 - 81$
5. $x^2 - 5$

Difference of two squares Solutions

Try factorising these expressions using the difference of two squares

1. $x^2 - 6^2$ $= (x-6)(x+6)$

2. $y^2 - 144$ $= (y+12)(y-12)$

3. $x^2 - y^2$ $= (x+y)(x-y)$

4. $4t^2 - 81$ $= (2t-9)(2t+9)$

5. $x^2 - 5$ $= \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)$

So far we have been factorising quadratic expressions where $a = 1$. For example $x^2 - 2x - 15$

Time to try some trickier quadratics!

Have a go at this one…

Factorise

$$6x^2 + 19x + 10$$

Factorise

$$6x^2 + 19x + 10$$

- If you got $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$ Well done!

Feeling confident? You can skip on to the Trickier Quadratics questions.

There are many methods for factorising quadratics where $a > 1$

- Follow this link to discover 'the grid method'.

Alternatively, if you want to refresh your memory on the method that you learnt at school - Search Tricky Quadratics to find a video to help you.

The grid method for $ax^2 + bx + c$

When using a grid we noticed the following:

Remember this from the Expanding Double Brackets section?

$(3x - 5)$

$(x + 3)$

$\times$	$3x$	-5
x	$3x^2$	$-5x$
$+3$	$+9x$	-15

The products of the diagonals are identical expressions

$$9x \times -5x = -45x^2$$
$$3x^2 \times -15 = -45x^2$$

The sum of these terms make the middle term in the simplified expression

$$3x^2 + 9x - 5x - 15$$

$$3x^2 + 4x - 15$$

We are now going to use this method to help us factorise quadratics where the x^2 coefficient is not 1

The grid method for $ax^2 + bx + c$

Let's start with our previous question.

Factorise

$$6x^2 + 19x + 10$$

We can put the $6x^2$ and the $+10$ straight into the grid as shown below

$\times$		
	$6x^2$	
		$+10$

The product of this diagonal is $60x^2$

$\times$		
	$6x^2$	?
	?	$+10$

So the product of this diagonal is also $60x^2$

Got some idea about what the missing terms might be already?

Remember their sum must also equal $19x$

The grid method for $ax^2 + bx + c$

Factorise

$$6x^2 + 19x + 10$$

$\times$		
	$6x^2$	?
	?	$+10$

The product of this diagonal is $60x^2$

The sum of this diagonal is $19x$

Pairs of factors that make 60?....

.... that also sum to give 19?

$\times$		
	$6x^2$	$4x$
	$15x$	$+10$

It doesn't matter which order you put $15x$ and $4x$ into the grid as multiplication is commutative.

The grid method for $ax^2 + bx + c$

Factorise

$$6x^2 + 19x + 10$$

Time to factorise in the grid!

$\times$	?	?
	$6x^2$	$4x$
	$15x$	$+10$

$\times$	$3x$	2
	$6x^2$	$4x$
	$15x$	$+10$

Find the Highest Common Factor (HCF) of each column and write it at the top

HCF of $6x^2$ and $15x$ is $3x$
HCF of $4x$ and 10 is 2

Have you got an idea about what we are going to do next?

Factorise

$$6x^2 + 19x + 10$$

Time to factorise the grid!

$\times$	$3x$	2
?	$6x^2$	$4x$
?	$15x$	$+10$

Find the Highest Common Factor of each row and write them on the left

HCF of $6x^2$ and $4x$ is $2x$
HCF of $15x$ and 10 is 5

$\times$	$3x$	2
$2x$	$6x^2$	$4x$
5	$15x$	$+10$

This means that $6x^2 + 19x + 10$ factorises to $(2x + 5)(3x + 2)$

Expanding is much quicker than factorising – so it is a good idea to expand $(2x + 5)(3x + 2)$ as a check.

Trickier Quadratics

- Try factorising these expressions
- You might want to try the grid method.

1. $3x^2 - 10x - 8$

2. $2x^2 - 7x + 6$

3. $4y^2 + 20y + 9$

4. $6x^2 - 13x - 8$

5. $20x^2 + x - 12$

*Hint. There are some partially filled grids on the next slide if you want to use them

Trickier Quadratics

For some help with factorising you can complete the grids by filling in the blanks

$\times$	$3x$	
	$3x^2$	$2x$
		-8

$$3x^2 - 10x - 8$$

$\times$		
$2x$	$2x^2$	
	$-3x$	$+6$

$$2x^2 - 7x + 6$$

$\times$		
	$4y^2$	
	$2y$	$+9$

$$4y^2 + 20y + 9$$

$\times$		
		$-3x$
		-8

$$6x^2 - 13x - 8$$

$\times$		
	$16x$	

$$20x^2 + x - 12$$

Trickier Quadratics Solutions

For some help with factorising you can complete the grids by filling in the blanks

$\times$	$3x$	2
x	$3x^2$	$2x$
-4	$-12x$	-8

$3x^2 - 10x - 8$

$= (3x + 2)(x - 4)$

$\times$	x	2
$2x$	$2x^2$	$-4x$
3	$-3x$	$+6$

$2x^2 - 7x + 6$

$= (2x - 3)(x - 2)$

$\times$	$2y$	9
$2y$	$4y^2$	$18y$
1	$2y$	$+9$

$4y^2 + 20y + 9$

$= (2y + 1)(2y + 9)$

$\times$	$3x$	-8
$2x$	$6x^2$	$-16x$
1	$3x$	-8

$6x^2 - 13x - 8$

$= (2x + 1)(3x - 8)$

$\times$	$5x$	4
$4x$	$20x^2$	$16x$
-3	$-15x$	-12

$20x^2 + x - 12$

$= (4x - 3)(5x + 4)$

Trickier Quadratics Solutions

1. $3x^2 - 10x - 8 \quad = (3x + 2)(x - 4)$

2. $2x^2 - 7x + 6 \quad = (2x - 3)(x - 2)$

3. $4y^2 + 20y + 9 \quad = (2y + 1)(2y + 9)$

4. $6x^2 - 13x - 8 \quad = (3x - 8)(2x + 1)$

5. $20x^2 + x - 12 \quad = (5x + 4)(4x - 3)$

Further Factorising Problems

These expressions are slightly different to the previous ones, but can still be factorised.

1. $2t^2 - 32$

2. $x^3 - 7x^2 + 12x$

3. $x^4 - x^2 - 2$

4. $y^4 - 625$

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Further Factorising Solutions

These expressions are subtly different to the previous ones, but can still be factorised.

1. $2t^2 - 32 \quad = 2(t^2 - 16) = 2(t-4)(t+4)$

2. $x^3 - 7x^2 + 12x \quad = x(x^2 - 7x + 12) = x(x-3)(x-4)$

3. $x^4 - x^2 - 2 \quad = (x^2 - 2)(x^2 + 1)$

4. $y^4 - 625 \quad = \underbrace{(y^2+5)(y^2-5)} = (y^2+5)\underbrace{(y-5)(y+5)}$

Difference of two squares – twice!

Without a calculator

What is the value of each of the following?

calculators not allowed

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

Hints available on the next slide

Without a calculator Hint

What is the value of each of the following?

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

- Can you factorise $9^2 - 1^2$?
- How does this help?

Without a calculator Solutions

Follow the link for the solutions

Still without a calculator

Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

Hints available on the next slide

Still without a calculator Hint

Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

It might seem strange advice but…..

- Replace 123 by n and 122 by $n-1$
- Now go on to factorise

Still without a calculator Solutions

Follow the link for the solutions

Top and Bottom

Simplify

$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

Hints available on the next slide

Top and Bottom Hint

Simplify

$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

- Factorise the numerator then the denominator
- What do you notice?

Top and Bottom Solution

Follow the link for the solutions

Still want more?

Explore the history of mathematics with this interactive historical timeline -in particular look for at Al-Khwarizmi. Can you find a famous artist and a mathematician whose triangle you met in the Expanding topic?

Discover how you can use factorising quadratics and apply it to higher powers by this neat trick shown in this nrich task.

Watch how you can apply difference of two squares to a fun numerical problem.