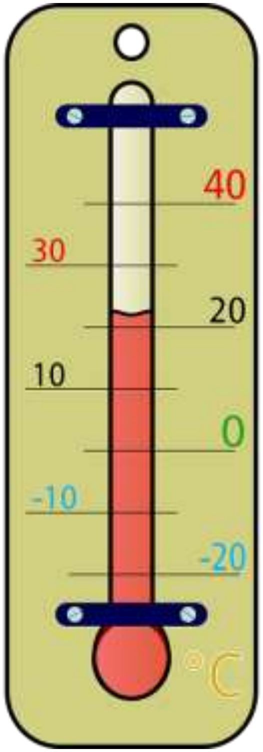
Advanced Mathematics Support Programme®

Did you know?

This is a well known formula that you might recognise.

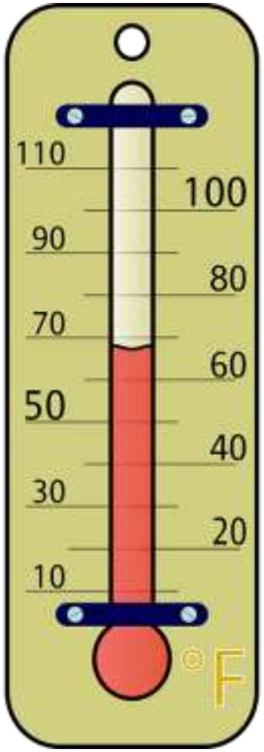


$$F = \frac{9}{5}C + 32$$

It is used to change temperatures in degrees Celsius ^{o}C to degrees Fahrenheit ^{o}F

For example: If it is 20^{o}C to find the temperature in ^{o}F you simply substitute C=20 into the formula above:

68^{o}F



What would I need to do if I wanted to convert from Fahrenheit to Celsius??

Rearranging 1

1. Solve $3x + 25 = 60$

2. Rearrange $z = w + 3$ to make w the subject

3. Rearrange $5x - 4 = 2y$ to make x the subject

4. Rearrange $y = \frac{t}{6}$ to make t the subject

5. $y = 6p^2 + 2$ rearrange to make p the subject

6. The area of a circle is found using $\mathrm{A} = \pi r^2$ Write the equation you would use to find the radius.

7. In a right angled triangle $sinx = \frac{Opp}{Hyp}$ write down the equation for finding the opposite side.

8. To change temperatures in Celsius to Fahrenheit this formula is used.
$$F = \frac{9}{5}C + 32$$
Rearrange to give the formula for converting Celsius to Fahrenheit

Rearranging 1

Solutions on the next slide….

Rearranging 1 Solutions

1. Solve $3x + 25 = 60$

$$3x = 60 - 25$$
$$3x = 35$$
$$x = \frac{35}{3}$$

2. Rearrange $z = w + 3$ to make w the subject

$$z - 3 = w$$
$$or\ w = z - 3$$

3. Rearrange $5x - 4 = 2y$ to make x the subject

$$5x = 2y + 4$$
$$x = \frac{2y + 4}{5}$$

4. Rearrange $y = \frac{t}{6}$ to make t the subject

$$6y = t$$
$$or\ t = 6y$$

Unsure about any of these? Search Rearranging Formulae. Next try Skills check 2….

Rearranging 1 Solutions

5. $y = 6p^2 + 2$ rearrange to make p the subject

$$y - 2 = 6p^2 \qquad p^2 = \frac{y-2}{6}$$

$$p = \pm\sqrt{\frac{y-2}{6}}$$

6. The area of a circle is found using $A = \pi r^2$ Write the equation you would use to find the radius.

$$\frac{A}{\pi} = r^2 \qquad r = \sqrt{\frac{A}{\pi}}$$

7. In a right angled triangle $sinx = \frac{Opp}{Hyp}$ write down the equation for finding the opposite side.

$$Opp = Hyp \times sinx$$

8. To change temperatures in Celsius to Fahrenheit this formula is used.

$$F = \frac{9}{5}C + 32$$

Rearrange to give the formula for converting Celsius to Fahrenheit

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$5(F - 32) = 9C$$

$$\frac{5}{9}(F - 32) = C$$

Rearranging 2

1. Make x the subject of $x - f = y + b$

2. Make y the subject $ty - x^2 = b$

3. Make c the subject $ac + d = m^2$

4. Make a the subject $x(a - e) = d$

5. Make y the subject $b(y - b) = b^2$

6. To find velocity, v, we use the formula

   $$v^2 = u^2 + 2as$$

   Rearrange to find s

7. The area of a sector of a circle is given by $A = \frac{\theta \pi r^2}{360}$ Express θ in terms of A, π and r

8. Make x the subject $m(y - x) = t$

Rearranging 2

Solutions on the next slide….

Rearranging 2 Solutions

1. Make x the subject of $x - f = y + b$ → $x = y + b + f$

2. Make y the subject $ty - x^2 = b$ →

$$ty = b + x^2$$
$$y = \frac{b + x^2}{t}$$

3. Make c the subject $ac + d = m^2$ →

$$ac = m^2 - d$$
$$c = \frac{m^2 - d}{a}$$

4. Make a the subject $x(a - e) = d$ →

$$xa - xe = d$$
$$xa = d + xe$$
$$a = \frac{d + xe}{x}$$

or

$$a - e = \frac{d}{x}$$
$$a = \frac{d}{x} + e$$

Can you see that these are equivalent?

Rearranging 2 Solutions

5. Make y the subject $b(y - b) = b^2$

$$by - b^2 = b^2$$
$$by = 2b^2$$
$$y = 2b$$

Solution

6. To find velocity, v, we use the formula
$$v^2 = u^2 + 2as$$
Rearrange to find s

$$v^2 - u^2 = 2as$$
$$s = \frac{v^2 - u^2}{2a}$$

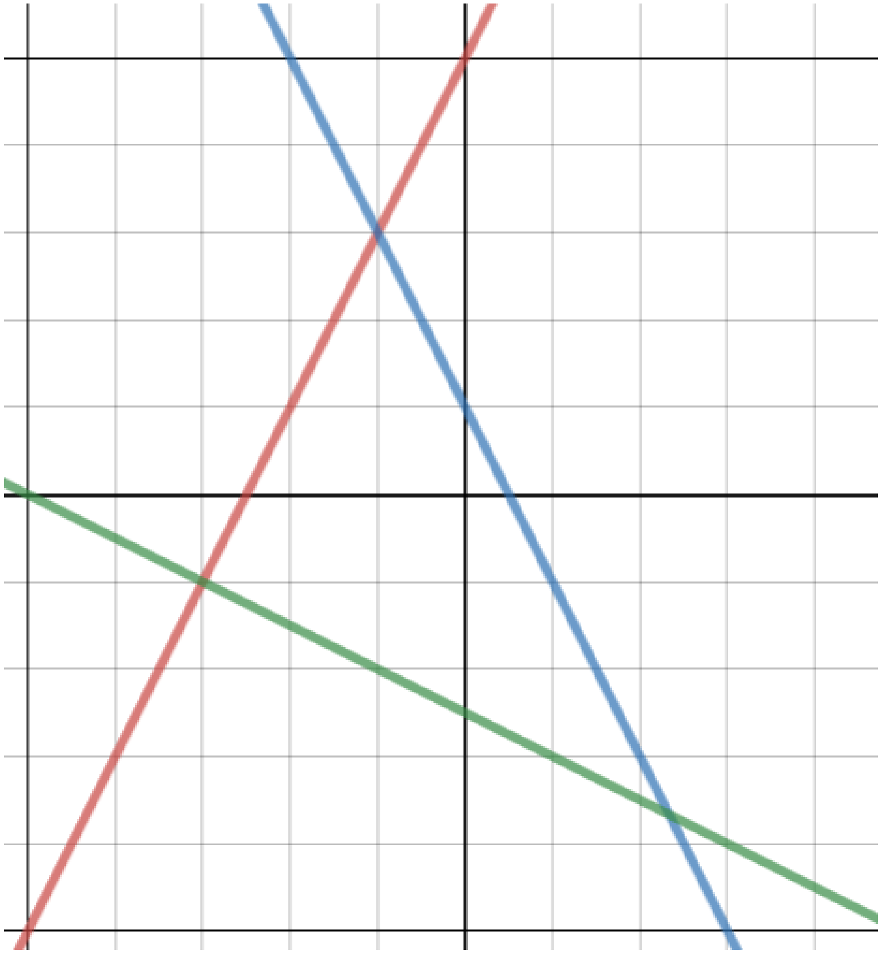
7. The area of a sector of a circle is given by $A = \frac{\theta \pi r^2}{360}$ Express θ in terms of A, π and r

$$360A = \theta \pi r^2$$
$$\theta \pi r^2 = 360A$$
$$\theta = \frac{360A}{\pi r^2}$$

8. Make x the subject $m(y - x) = t$

$$my - mx = t$$
$$my = t + mx$$
$$mx = my - t$$
$$x = \frac{my - t}{m}$$

Line them up 1



Which is which?

- $y = 2x + 5$
- $2y + x + 5 = 0$
- $y + 2x = 1$

How does rearranging enable you to justify your answer?

Line them up 1 Solution

Which is which?

- $y = 2x + 5$
- $2y + x + 5 = 0$
- $y + 2x = 1$

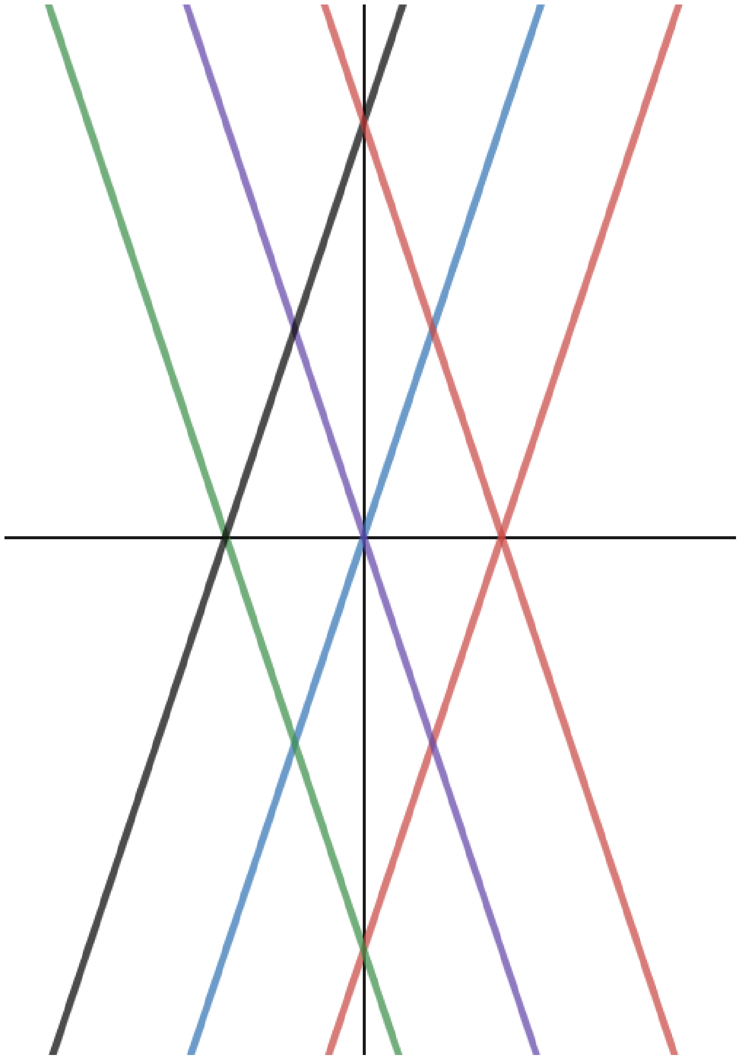
Why?

- $y = 2x + 5$
- $y = -\frac{x}{2} - \frac{5}{2}$
- $y = -2x + 1$

By rearranging into the form $\boldsymbol{y = mx + c}$ you can easily compare the gradient and intercept of each line.

Line them up 2

Label the lines with these equations.



$$y = 4 - 3x$$

$$y + 3x + 4 = 0$$

$$y + 3x = 0$$

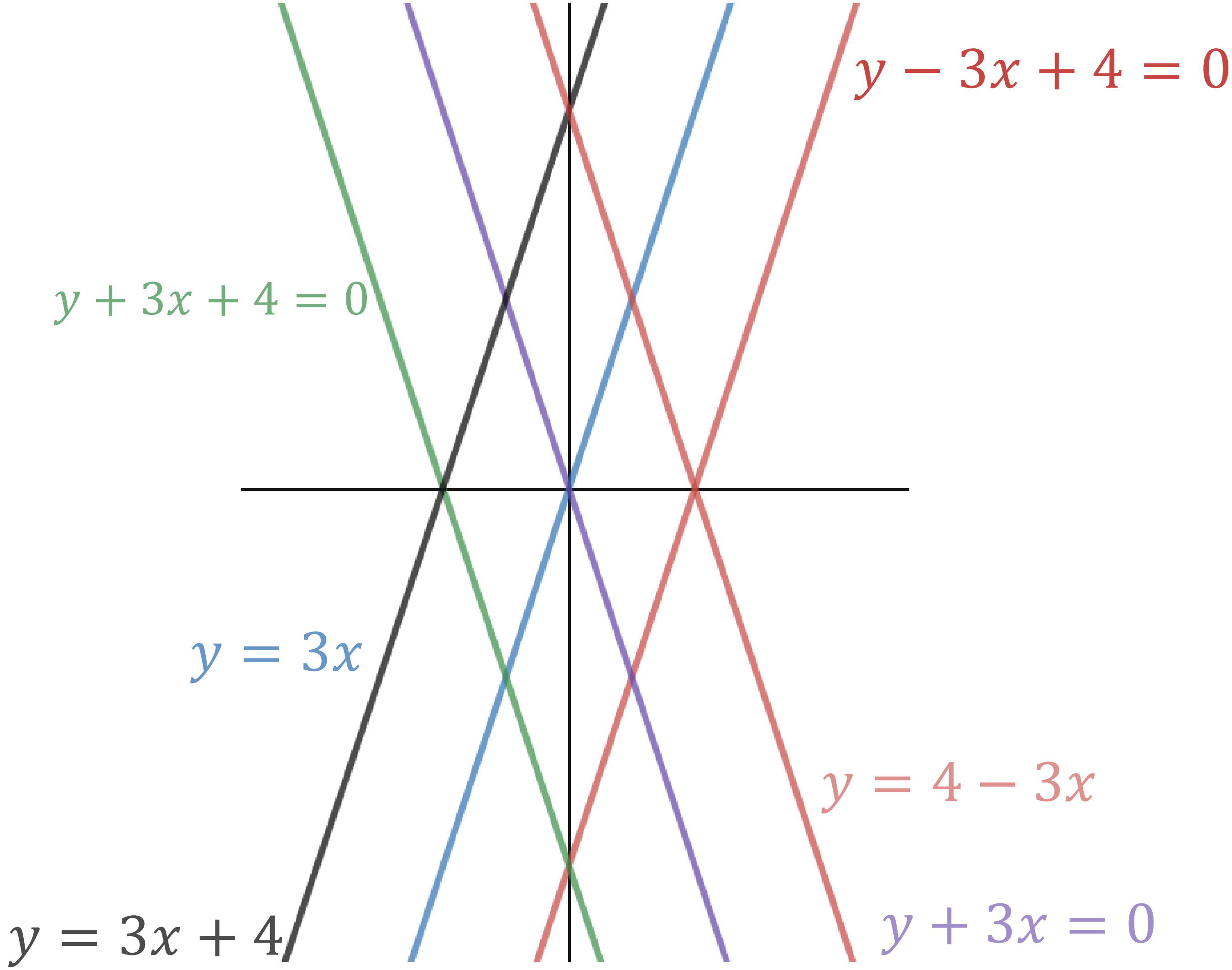
$$y = 3x$$

$$y = 3x + 4$$

$$y - 3x + 4 = 0$$

Line them up 2 Solution

Label the lines with these equations.



Pairing up

Can you sort the cards into pairs under the following headings:

- These lines are perpendicular
- These lines have the same x intercept
- These lines have the same y intercept
- These lines are parallel
- These lines go through the point (1,5)
- These lines…

$3y = 2x - 8$	$y = -(x + 8)$	$y = 4x + 4$
$2y + x = 4$	$y = 6x - 4$	$y = 8x - 3$
$y + x + 8 = 0$	$2y = 8x + 3$	$4y = x + 3$
$2y + 8 = 3x$	$y + 6x = 11$	$y + 4x + 6 = 0$

Pairing up Solution

Can you sort the cards into pairs under the following headings:

- These lines are perpendicular

$4y = x + 3$	$y + 4x + 6 = 0$

- These lines are parallel

$y = 4x + 4$	$2y = 8x + 3$

- These lines have the same y intercept

$2y + 8 = 3x$	$y = 6x - 4$

- These lines have the same x intercept

$2y + x = 4$	$3y = 2x - 8$

- These lines go through the point (1,5)

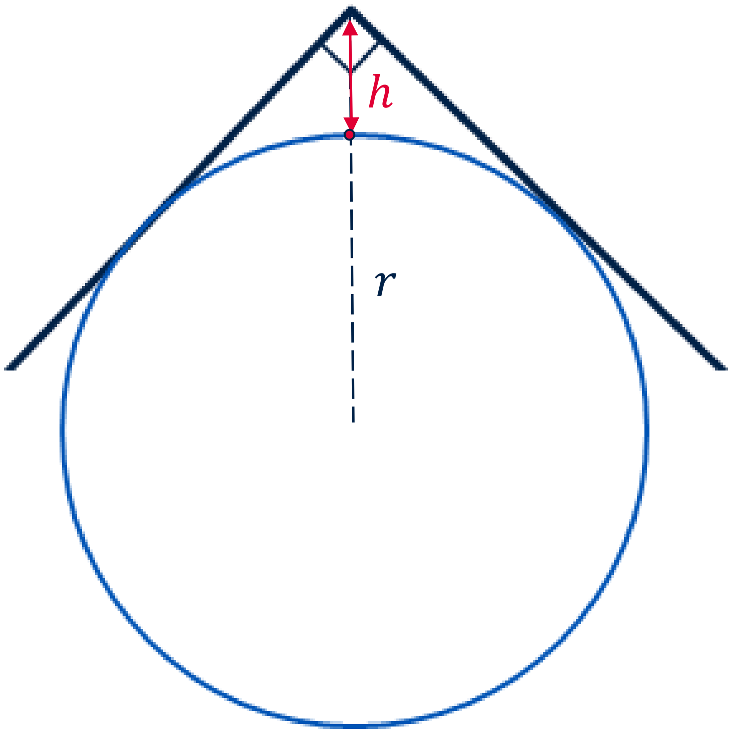
$y = 8x - 3$	$y + 6x = 11$

- These lines are the same line

$y + x + 8 = 0$	$y = -(x + 8)$

Pipe Problem

Can you find the radius of the pipe shown if the only measurement you can take is the one marked h?



Click here to watch a video with hints and the solution

A function relates an input to an output

Here is an example of a function machine

$$\text{Input} \longrightarrow \boxed{\times 3} \longrightarrow \boxed{+2} \longrightarrow \text{Output}$$

Complete the following table for the function machine shown

$$\longrightarrow \boxed{\times 3} \longrightarrow \boxed{+2} \longrightarrow$$

Input	Output
5	
-4	
x	
	17
	x

What do you notice?

A function relates an input to an output

$$\longrightarrow \boxed{\times 3} \longrightarrow \boxed{+2} \longrightarrow$$

Input	Output
5	17
-4	-10
x	$3x+2$
5	17
$\frac{x-2}{3}$	x

An inverse function goes the other way

To reverse the process inverse operations are used.

Output $\longleftarrow \boxed{\div 3} \longleftarrow \boxed{-2} \longleftarrow$ Input

Important! The inverse should give us back the original value

Let's introduce function notation that you will use in A level maths:

$f(5) = 3 \times 5 + 2 = 17$

Input	Output
5	17
-4	-10
x	$3x+2$
$\leftarrow \div 3 \leftarrow -2 \leftarrow$	
5	17
$\frac{x-2}{3}$	x

$\longrightarrow f(5) = 3 \times 5 + 2 = 17$

$\longrightarrow f(-4) = 3 \times -4 + 2 = -10$

$\longrightarrow \boxed{f(x) = 3x + 2}$

An inverse function goes the other way

$\longrightarrow f^{-1}(17) = (17-2)/3 = 5$

$\longrightarrow \boxed{f^{-1}(x) = \frac{x-2}{3}}$

Important! The inverse should give us back the original value

Lets check: $f(5) = 17 \; and \; f^{-1}(17) = 5$

Rearranging and Functions

Original function

$$f(x) = 3x + 2$$

Inverse function

$$f^{-1}(x) = \frac{x-2}{3}$$

Find the inverse of each of these functions.

1. $f(x) = 3x - 5$
2. $f(x) = 4x + 7$
3. $f(x) = \frac{x}{2} + 1$
4. $f(x) = \frac{x+2}{3}$
5. $f(x) = \frac{2}{3}x + 3$
6. $f(x) = 3 - 2x$

Instead of reversing a function machine - try re-arranging the original function to make x the subject

Rearranging and Functions

Solutions on the next slide….

Rearranging and Functions

Find the inverse of each of these functions.

1. $f(x) = 3x - 5$ $\qquad f^{-1}(x) = \dfrac{x+5}{3}$

2. $f(x) = 4x + 7$ $\qquad f^{-1}(x) = \dfrac{x-7}{4}$

3. $f(x) = \dfrac{x}{2} + 1$ $\qquad f^{-1}(x) = 2(x-1)$

*Be careful not to get the notation f^{-1} mixed up with reciprocals and negative powers!

Find the inverse of each of these functions.

4. $f(x) = \dfrac{x+2}{3}$ $\qquad f^{-1}(x) = 3x - 2$

5. $f(x) = \dfrac{2}{3}x + 3$ $\qquad f^{-1}(x) = \dfrac{3(x-3)}{2}$

6. $f(x) = 3 - 2x$ $\qquad f^{-1}(x) = \dfrac{3-x}{2}$

If you want to explore functions further then click here.

Still want more?

Read – Ten key reasons why developing algebraic skills is so important!

Discover more about the graphs of a function and its inverse by exploring this GeoGebra activity.

Watch and learn how maths, in particular the correct use of brackets, influences music, poetry and even rap!

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