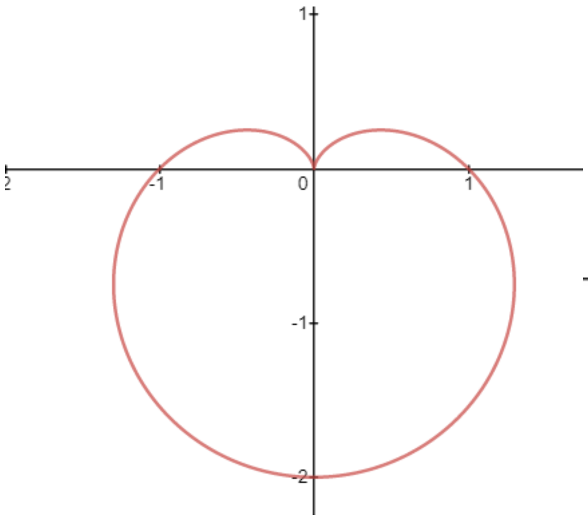
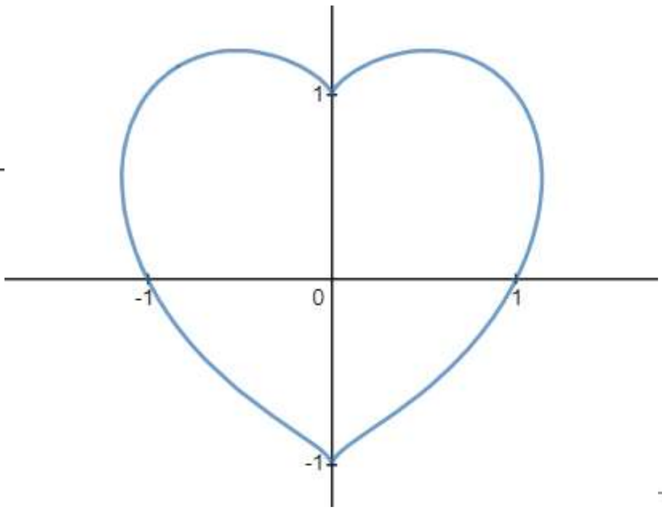
Advanced Mathematics Support Programme®

Did you know?

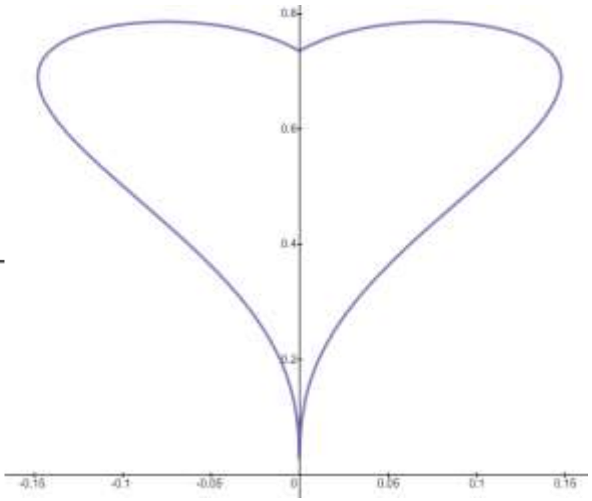
- Being able to express equations in different forms gives us different information
- Later we'll be looking at information needed to sketch graphs
- If you continue your maths studies to A Level Further Maths, you will draw graphs such as these



$r = 1 - sin\theta$



$(x^2 + y^2 - 1)^3 - x^2y^3 = 0$



$x = \sin(t)\cos(t)\log(|t|)$
$y = |t|^{0.3}\sqrt{\cos(t)}$

Further Factorising 1

1. The equation of a line is given as

$$3y + 4x - 2 = 0.$$

What is the gradient of the line?

2. A rectangle has area A, length y and width $x - 2$. Write an expression for the length of the rectangle, y, in terms of A and x

3. Make x the subject of:

$$ax - y = z + bx$$

4. Make b the subject of:

$$5(b - p) = 2(b + 3)$$

5. John says the first step to rearranging $\frac{x-a}{f} = 3g$ is to add $a\ to\ 3g$. Is he right? Explain your answer.

6. Make a the subject of

$$5(a - t) = 3(a + x)$$

7. Make x the subject of

$$ay + x = 4x + xb$$

8. Make x the subject of

$$2\pi\sqrt{x + t} = 4$$

Further Factorising 1

Solutions on the next slide….

Further Factorising 1 Solutions

1. The equation of a line is given as

$$3y + 4x - 2 = 0.$$

What is the gradient of the line?

$$3y = -4x + 2$$
$$y = -\frac{4}{3}x + \frac{2}{3}$$
gradient = $-\frac{4}{3}$

2. A rectangle has area A, length y and width $x - 2$. Write an expression for the length of the rectangle, y, in terms of A and x

$$A = y(x - 2)$$
$$y = \frac{A}{x - 2}$$

3. Make x the subject of:

$$ax - y = z + bx$$

$$ax - bx = z + y$$
$$x(a - b) = z + y$$
$$x = \frac{z + y}{a - b}$$

4. Make b the subject of:

$$5(b - p) = 2(bx + 3)$$

$$5b - 5p = 2bx + 6$$
$$5b - 2bx = 6 + 5p$$
$$b(5 - 2x) = 6 + 5p$$
$$b = \frac{6 + 5p}{5 - 2x}$$

Unsure about any of these? Search Rearranging and Factorising Next try Skills check 2….

Further Factorising 1 Solutions

5. John says the first step to rearranging $\frac{x-a}{f} = 3g$ is to add $a\ to\ 3g$. Is he right? Explain your answer.

No, the first step is to multiply by f

6. Make a the subject of

$$5(a - t) = 3(a + x)$$

$$5a - 5t = 3a + 3x$$
$$5a - 3a = 3x + 5t$$
$$2a = 3x + 5t$$
$$a = \frac{3x + 5t}{2}$$

7. Make x the subject of

$$ay + x = 4x + xb$$

$$ay = 3x + xb$$
$$x(3 + b) = ay$$
$$x = \frac{ay}{3 + b}$$

8. Make x the subject of

$$2\pi\sqrt{x + t} = 4$$

$$\sqrt{x + t} = \frac{4}{2\pi}$$
$$\sqrt{x + t} = \frac{2}{\pi}$$
$$x + t = \frac{4}{\pi^2} \qquad x = \frac{4}{\pi^2} - t$$

Further Factorising 2

1. Make y the subject of

$$xy + 6 = 7 - ky$$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

4. Make y the subject of $\dfrac{4}{y} + 1 = 2x$

5. Displacement can be expressed as

$$s = ut + \frac{1}{2}at^2$$

Express a in terms of s, u and t

6. Make y the subject of $\sqrt{by^2 - x} = D$

7. The area of a trapezium has formula

$$A = \frac{1}{2}\left(\frac{a+b}{h}\right)$$

Express h in terms of A, a and b

8. Make t the subject $b(t + a) = x(t + b)$

Further Factorising 2

Solutions on the next slide….

Further Factorising 2 Solutions

1. Make y the subject of

$$xy + 6 = 7 - ky$$

$$xy + ky = 1$$
$$y(x + k) = 1$$
$$y = \frac{1}{x + k}$$

2. Find an expression for the area of a rectangle with length, $(y - x)$ and width, $(x - 2)$

Area = $(y - x)(x - 2)$

$$A = xy - x^2 - 2y + 2x$$

3. Rewrite your expression in Q2 to have y expressed in terms of A and x

$$A = xy - x^2 - 2y + 2x$$
$$2y - xy = 2x - x^2 - A$$
$$y(2 - x) = 2x - x^2 - A$$
$$y = \frac{2x - x^2 - A}{2 - x}$$

4. Make y the subject of $\frac{4}{y} + 1 = 2x$

$$\frac{4}{y} = 2x - 1$$
$$y(2x - 1) = 4$$
$$y = \frac{4}{2x - 1}$$

Further Factorising 2 Solutions

5. Displacement can be expressed as

$$s = ut + \frac{1}{2}at^2$$

Express a in terms of s, u and t

$$\frac{1}{2}at^2 = s - ut$$
$$at^2 = 2s - 2ut$$
$$a = \frac{2s - 2ut}{t^2}$$

Solution

6. Make y the subject of $\sqrt{by^2 - x} = D$

$$by^2 - x = D^2$$
$$by^2 = D^2 + x$$
$$y^2 = \frac{(D^2 + x)}{b}$$
$$y = \pm\sqrt{\frac{D^2 + x}{b}}$$

7. The area of a trapezium has formula

$$A = \frac{1}{2}\left(\frac{a+b}{h}\right)$$

Express h in terms of A, a and b

$$2hA = a + b$$
$$h = \frac{a + b}{2A}$$

8. Make t the subject $b(t + a) = x(t + b)$

$$bt + ba = xt + xb$$
$$bt - xt = xb - ba$$
$$t(b - x) = xb - ba$$
$$t = \frac{xb - ba}{b - x}$$

Equivalent quadratics

Sort the expressions below in to 4 sets of 4 equivalent expressions.

$x^2 - 25$	$2x^2 - 2$
$(x+5)(x+6) - x - 55$	$(x+5)(x-5)$
$2(x^2 - 1)$	$(x+5)^2 - 10x - 50$
$2(x+3)(x-1)$	$2(x+1)(x-1)$
$(x+5)^2 - 50$	$2(x+2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x+5)(x-5) + 10x$
$2(x+1)^2 - 8$	$(x-5)(x+6) - x + 5$
$x^2 + 10x - 25$	$2(x+1)^2 - 4(x+1)$

Equivalent quadratics

Solutions on the next slide….

Equivalent quadratics Solution

Sort the expressions below in to 4 sets of 4 equivalent expressions.

$x^2 - 25$	$2x^2 - 2$
$(x + 5)(x + 6) - x - 55$	$(x + 5)(x - 5)$
$2(x^2 - 1)$	$(x + 5)^2 - 10x - 50$
$2(x + 3)(x - 1)$	$2(x + 1)(x - 1)$
$(x + 5)^2 - 50$	$2(x + 2)^2 - 4x - 14$
$2x^2 + 4x - 6$	$(x + 5)(x - 5) + 10x$
$2(x + 1)^2 - 8$	$(x - 5)(x + 6) - x + 5$
$x^2 + 10x - 25$	$2(x + 1)^2 - 4(x + 1)$

Mean squares

- Take two positive values greater than 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

Hint on next slide if you need it!

Mean squares

- Take two positive values greater than 1
- Find the mean of the two values
- Square it

Then

- Take the same two values
- Square them
- Find the mean of the squares

Which value is greater?

Is this always true?

Can you prove it?

- Try out several examples
- Is one expression always bigger than the other?
- Next try using x and y instead.
- If you subtract one expression from the other, can you work out if it's positive or negative?

Mean squares

Follow the link to the solutions

Difference of numeric squares

Problem 1:

Mrs Gryce was asked to calculate 18×12 by Mr Lo who had forgotten his calculator and was doing some marking.

Mrs Gryce quickly responded

“Well, that’s just $15^2 - 9$ which is 216”

Mr Lo was amazed.

- How did she know so quickly what the answer was?

Problem 2:

- Use the fact that 3 x 4 = 12
- Can you quickly work out a value for $(3.5)^2$?

Can you see a connection between the previous question and this one?

Difference of numeric squares

Follow the link to the solutions

The Quadratic Formula

We've all used the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- But where does it come from?

- Can you prove why the quadratic formula works?

The next activity is all about doing just that!

The Quadratic Formula

Rearrange these steps in order to prove the quadratic formula

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\left(x + \frac{b}{2a}\right) = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = -\frac{c}{a}$	$ax^2 + bx = -c$
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	$\left(x + \frac{b}{2a}\right) = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

There are written steps to help you on the next slide if you'd like them

The Quadratic Formula

Match the steps below with the algebra on the previous slide for a slightly easier version

Step 1: Subtract c from both sides

Step 2: Divide both sides by a

Step 3: Complete the square on the left hand side

Step 4: Add $\frac{b^2}{4a^2}$ to both sides

Step 5: Make the right hand side into a single expression

Step 6: Take the square root of both sides

Step 7: Simplify the denominator on the right hand side

Step 8: Subtract $\frac{b}{2a}$ from both sides

Step 9: You now have the quadratic formula!

For a challenge – just use the algebra!

The Quadratic Formula - Rearranging

Follow the link to the solutions

Equations of …..?

You should have come across an equation like this in your GCSE course:

$$x^2 + y^2 = 25$$

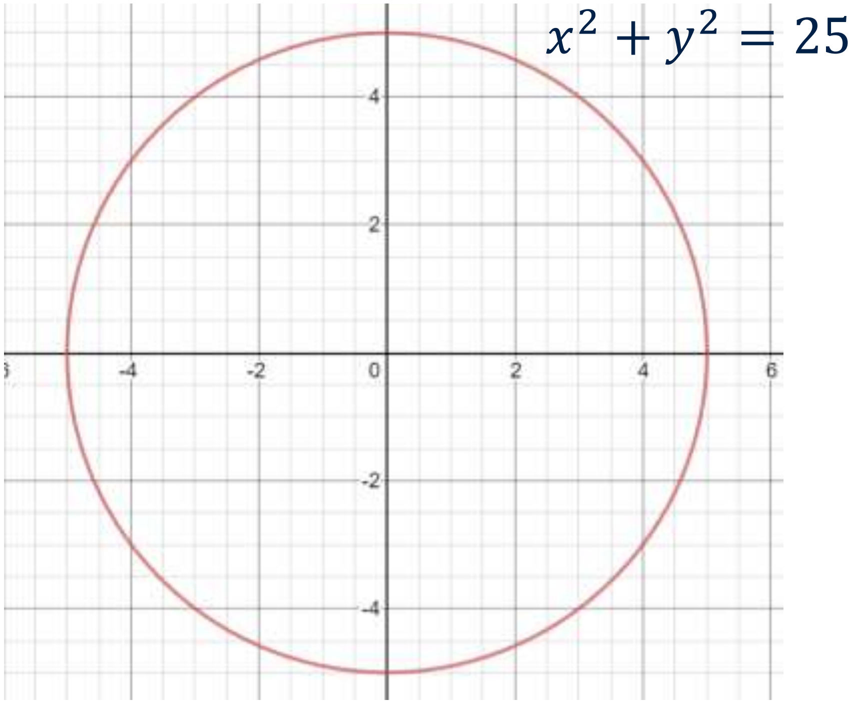
- Can you remember what this would look like?
- Can you describe it?

You could draw this on DESMOS https://www.desmos.com/calculator

Equations of Circles

$$x^2 + y^2 = 25$$

Represents a circle with centre (0,0) and radius 5



Generally the equation of a circle with centre (0,0) and radius r can be written as

$$x^2 + y^2 = r^2$$

But what happens if the centre is not (0,0) ?

Equations of .....?

Let's have a look at this equation

$$\boxed{x^2 + 4x + y^2 - 6y = 12}$$

We can rearrange this by completing the square separately for the x terms and y terms

$$x^2 + 4x = (x+2)^2 - 4 \text{ and } y^2 - 6y = (y-3)^2 - 9$$

So

$$x^2 + 4x + y^2 - 6y = 12$$

Can be written as

$$(x+2)^2 - 4 + (y-3)^2 - 9 = 12$$

$$(x+2)^2 + (y-3)^2 - 13 = 12$$

Collect like terms

$$(x+2)^2 + (y-3)^2 = 25$$

Rearrange to become

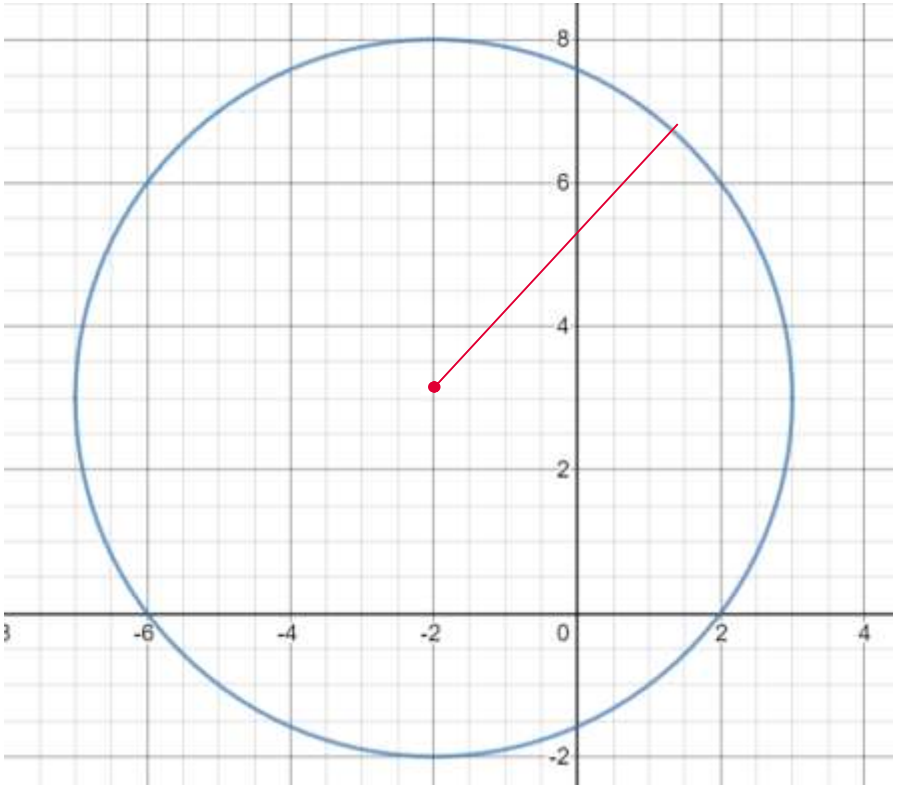
What do you think this equation represents?

Equations of circles

$$(x+2)^2+(y-3)^2 = 25$$

Represents a circle with
Centre (-2,3)
Radius 5

$(x+2)^2+(y-3)^2 = 25$



If we expand this equation

$$(x+2)^2 + (y-3)^2 = 25$$
$$x^2+4x+4+y^2-6y+9=25$$
$$x^2+4x+y^2-6y+13=25$$
$$x^2+4x+y^2-6y=12$$

We return it to the original form

Equations of circles

- Can you find the centre and radii of these circles by rearranging into the form

$$(x + a)^2 + (y - b)^2 = r^2$$

$x^2 - 8x + y^2 - 2y = 19$	$x^2 + 6x + y^2 - 10y = 15$

Equations of circles

A

Follow the link to the solutions

Still want more?

Discover how using a graphing app such as Desmos or GeoGebra can help you gain insight into circles, tangents and graphs in general. Gain skills useful for A level maths.

Watch a TED talk from Dr Hannah Fry which trys to answer the question “Is life too complex?” You will see that you can actually write equations that model human behaviour!

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