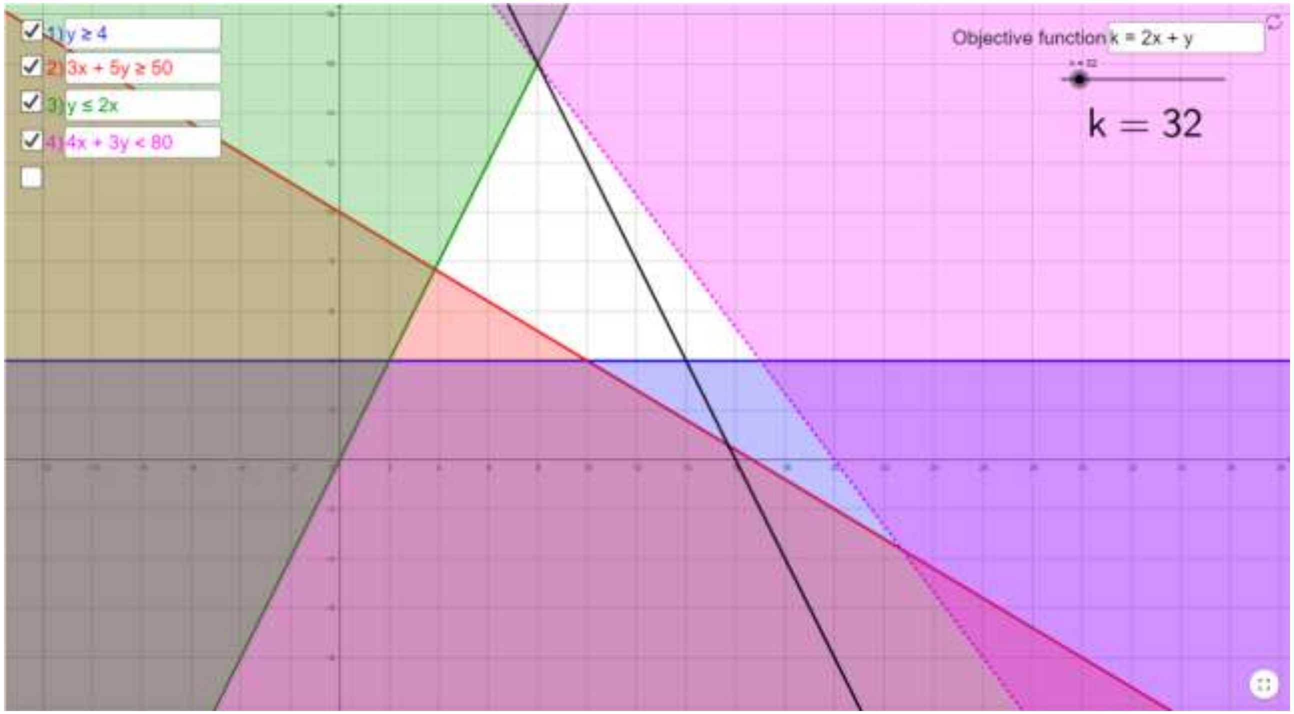
Advanced Mathematics Support Programme®

Did you know?

Linear programming is a method that involves solving a set of linear equations or inequalities in order to find the best solution.



It is very useful in industry for finding the best level of production, or the maximum profit depending on varying costs, sales, mix of products or availability of labour etc…

Solving Linear 1

Solve the following:

1. $8x - 3 = 5x + 13$

2. $3x + 1 > 10$ and $2x + 7 < 15$

3. $3(x + 6) > 12$

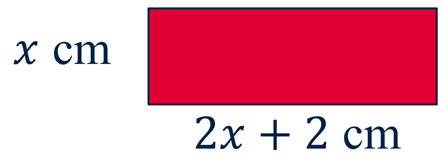
4. $24 - 3x = 9$

5. $14 \geq 8 + 5x$

6. $6 - 2x < 5x + 34$

7. $\dfrac{2x + 3}{7} = \dfrac{4x - 5}{3}$

8. The perimeter of the rectangle is 24cm. Find the value of x



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Solving Linear 1

Solutions on the next slide….

Solving Linear 1 Solutions

1. $8x - 3 = 5x + 13$ →

$$3x - 3 = 13$$
$$3x = 16$$
$$x = \frac{16}{3}$$

2. $3x + 1 > 10$
and $2x + 7 < 15$ →

$$3x > 9 \qquad 2x < 8$$
$$x > 3 \qquad x < 4$$

So $3 < x < 4$

3. $3(x + 6) > 12$ →

$$x + 6 > 4 \quad \text{or} \quad 3x + 18 > 12$$
$$x > -2 \qquad 3x > -6$$
$$x > -2$$

4. $24 - 3x = 9$ →

$$-3x = -15$$
$$x = 5$$

Unsure about any of these? Search Solving linear equations. Next try Solving Linear 2 …

Solving Linear 1 Solutions

5. $14 \geq 8 + 5x$

$$6 \geq 5x$$
$$\frac{6}{5} \geq x \text{ or } x \leq \frac{6}{5}$$

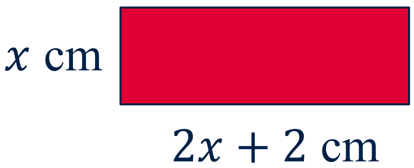
6. $6 - 2x < 5x + 34$

$$6 < 7x + 34$$
$$-28 < 7x$$
$$-4 < x \text{ or } x > -4$$

7. $\dfrac{2x+3}{7} = \dfrac{4x-5}{3}$

$$3(2x+3) = 7(4x-5)$$
$$6x + 9 = 28x - 35$$
$$44 = 22x$$
$$x = 2$$

8. The perimeter of the rectangle is 24cm. Find the value of x



$$x + (2x+2) + x + (2x+2) = 24$$
$$6x + 4 = 24$$
$$6x = 20$$
$$x = \frac{10}{3}$$

Unsure about any of these? Search Solving linear equations. Next try Solving Linear 2 …

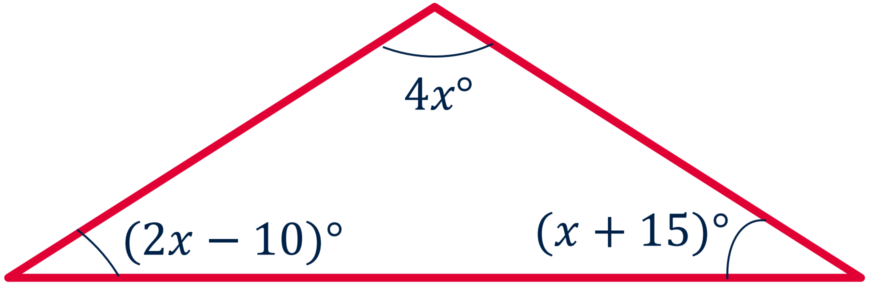
Solving Linear 2

Solve the following:

1. $6x + 5 = 47$
2. $5x + 7 = x + 25$
3. $7(x - 4) = 14$
4. $29 - 4x < 22$
5. $3x < 2x - 1 < 4x + 2$

   Hint: Split into two inequalities

6. $19 + 2x = 3x + 15$
7. $\dfrac{3x-1}{5} \geq \dfrac{3x+5}{2}$
8. Find the value of x



Solving Linear 2

Solutions on the next slide….

Solving Linear 2 Solutions

Solve the following:

1. $6x + 5 = 47$

$$6x = 42$$
$$x = 7$$

2. $5x + 7 = x + 25$

$$4x + 7 = 25$$
$$4x = 18$$
$$x = 4.5$$

3. $7(x - 4) = 14$

$$x - 4 = 2 \qquad \text{or} \qquad 7x - 28 = 14$$
$$x = 6 \qquad\qquad 7x = 42$$
$$x = 6$$

4. $29 - 4x < 22$

$$29 - 22 < 4x \qquad \text{or} \qquad -4x < -7$$
$$7 < 4x \qquad\qquad x > \frac{-7}{-4}$$
$$\frac{7}{4} < x \qquad\qquad x > \frac{7}{4}$$

When multiplying or dividing by a negative, the inequality reverses direction

Solving Linear 2 Solutions

5. $3x < 2x - 1 < 4x + 2$

$$3x < 2x - 1$$
$$x < -1$$

$$2x - 1 < 4x + 2$$
$$-1 < 2x + 2$$
$$-3 < 2x$$
$$-\frac{3}{2} < x$$

So $-\frac{3}{2} < x < -1$

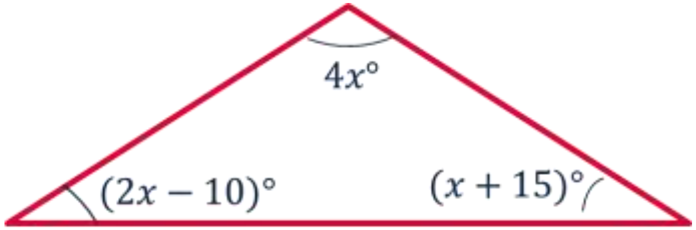
6. $19 + 2x = 3x + 15$

$$19 = x + 15$$
$$4 = x$$

7. $\dfrac{3x - 1}{5} \geq \dfrac{3x + 5}{2}$

$$2(3x - 1) \geq 5(3x + 5)$$
$$6x - 2 \geq 15x + 25$$
$$-2 \geq 9x + 25$$
$$-27 \geq 9x$$
$$-3 \geq x$$

8. Find the value of x



$$4x + (2x - 10) + (x + 15) = 180$$
$$7x + 5 = 180$$
$$7x = 175$$
$$x = 25$$

Piggy in the middle

The number in the middle of each group of 3 adjoining cells is the average of its two neighbours.

5			**23**	

What number goes in the right hand cell?

Piggy in the middle Solution

The number in the middle of each group of 3 adjoining cells is the average of its two neighbours.

What number goes in the right hand cell?

5			**23**	

As with most problems it is a good idea to begin by trying some numbers in the problem and see if that gives you a greater understanding.

In order for 23 to be the average of the numbers in the cells either side they must both be the same distance from 23 e.g.

16	**23**	30
23–7		23+7

Extending that to the other cells would mean:

5	9	16	**23**	30
	16–7		16+7	

BUT 9 is not the average of 5 and 16!

Let's generalise to try and form an equation:

$23-3x$	$23-2x$	$23-x$	**23**	$23+x$
	$+x$	$+x$	$+x$	$+x$

There are a few different equations that we could form…

Here is one example. $23-3x=5$

We can solve to get: $18=3x$

$x=6$

5	11	17	**23**	29

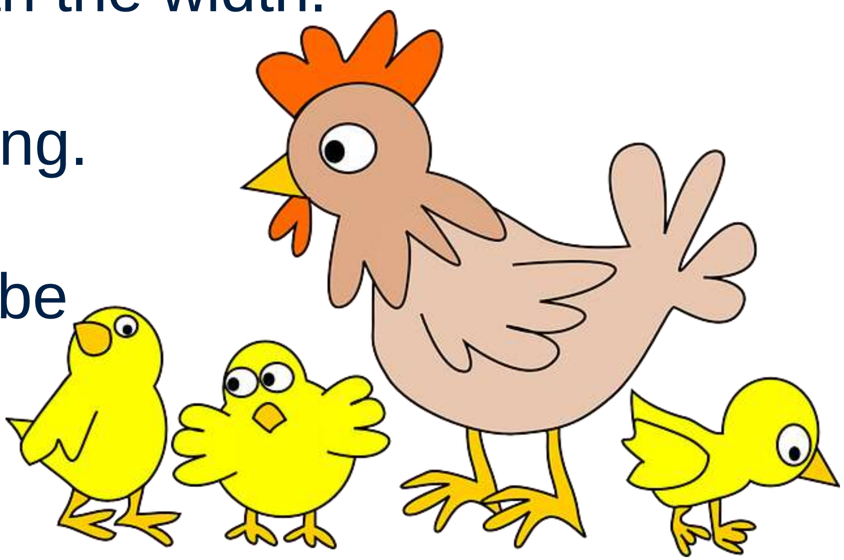
Chicken Run

Victoria has just bought some chickens. She wants to keep them safe in a small enclosure.

The enclosure will be a rectangle where the length is 3m longer than the width.

Victoria has only got 30m of fencing.

The area of the enclosure has to be greater than 20m^2

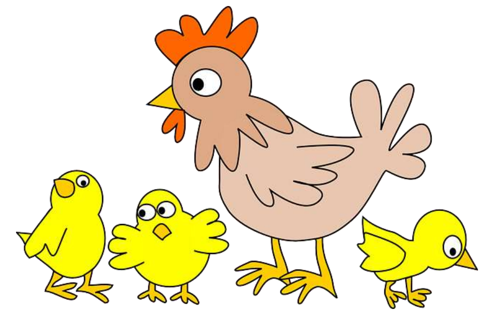


The length and width are integers.

How many different size enclosures can Victoria make?

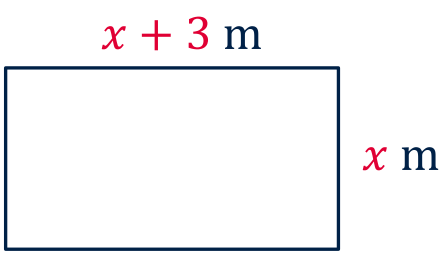
Chicken Run Solution

- The enclosure will be a rectangle
- The length is 3m longer than the width.
- Victoria has only got 30m of fencing.
- The area of the enclosure has to be greater than 20m^2
- The length and width are integers.



How many different size enclosures can Victoria make?

Draw a diagram



Write an inequality for the maximum perimeter of the fence

$$2(x+3) + 2x \leq 30$$
$$4x + 6 \leq 30$$
$$4x \leq 24$$
$$x \leq 6$$

The maximum width of the enclosure

Consider the area of the enclosure for different possible lengths and widths

Width (m)	Length (m)	Area (m^2)
6	9	54
5	8	40
4	7	28
3	6	18

Why have we not considered widths of 2 and 1?

So Victoria can make 3 different sized enclosures with an area greater than 20m^2

Crack the code

Can you decode this message?

		12	1	4		7	5	3	12	4	2	5								
7	4	3	3	6	15	4		9	2	6	9	8	4	10						

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

$$\frac{4r}{d-4}+\frac{2h}{s}=2$$

$$\frac{g-9}{y+4}=\frac{2}{3}$$

$$3rh+m=13$$

$$\frac{4g}{5}=12$$

$$\frac{2c-5+3(c-2)}{2c-1}=2$$

$$e^3<72$$

$$\frac{s+3y}{8s}=\frac{3}{4}$$

$$\frac{6k}{s}-5=11$$

$$100<t^2<169$$

$$\frac{8}{3a}=\frac{4}{a+3}$$

$$\frac{6r+8}{y}=4$$

$$2(3m+4)=7m+1$$

Hint available on next slide

Crack the code Hint

Can you decode this message?

12 1 4 7 5 3 12 4 2 5
7 4 3 3 6 15 4 9 2 6 9 8 4 10

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

Solve the equations in the following order:

1. $\frac{4g}{5} = 12$

2. $\frac{2c - 5 + 3(c-2)}{2c - 1} = 2$

3. $\frac{8}{3a} = \frac{4}{a+3}$

4. $\frac{g-9}{y+4} = \frac{2}{3}$

5. $\frac{6r+8}{y} = 4$

6. $2(3m+4) = 7m + 1$

7. $\frac{s+3y}{8s} = \frac{3}{4}$

8. $\frac{6k}{s} - 5 = 11$

9. $3rh + m = 13$

10. $\frac{4r}{d-4} + \frac{2h}{s} = 2$

11. $100 < t^2 < 169$

12. $e^3 < 72$

Crack the code Solution

Can you decode this message?

12 1 4   7 5 3 12 4 2 5

7 4 3 3 6 15 4   9 2 6 9 8 4 10

Solve the equations in the boxes below. Each letter will have a different positive integer solution between 0 and 16.

$\frac{4g}{5} = 12$ $g = 15$	$\frac{2c - 5 + 3(c-2)}{2c - 1} = 2$ $c = 9$	$\frac{8}{3a} = \frac{4}{a+3}$ $a = 6$
$\frac{g - 9}{y + 4} = \frac{2}{3}$ $y = 5$	$\frac{6r + 8}{y} = 4$ $r = 2$	$2(3m + 4) = 7m + 1$ $m = 7$
$\frac{s + 3y}{8s} = \frac{3}{4}$ $s = 3$	$\frac{6k}{s} - 5 = 11$ $k = 8$	$3rh + m = 13$ $h = 1$
$\frac{4r}{d - 4} + \frac{2h}{s} = 2$ $d = 10$	$100 < t^2 < 169$ $10 < t < 13$	$e^3 < 72$ $e < 4.16..$

"The mystery message cracked"

Linear Simultaneous Equations

There are two main ways to solve simultaneous equations.

Elimination

$$3x + 2y = 9$$
$$5x - 2y = -1$$

Add the two equations together to eliminate y

$$8x = 8$$
$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$3 \times 1 + 2y = 9$$
$$3 + 2y = 9$$
$$2y = 6$$
$$y = 3$$

Substitution

$$y + 3x = 5$$
$$2y + 7x = 11$$

Rearrange the first equation in terms of y and then substitute into the second equation

$$2(5 - 3x) + 7x = 11$$
$$10 - 6x + 7x = 11$$
$$x = 1$$

Now we have a value for x we can put it into one of the original equations to find y

$$y + 3 \times 1 = 5$$
$$y + 3 = 5$$
$$y = 2$$

Which method is best and when?

Simultaneous Equations

Solve the following:

1. $2x + y = 7$
   $2x - y = 1$

2. $3x + 2y = 7$
   $3x + 5y = 4$

3. $y = 4x + 3$
   $3x + 2y = 28$

4. $4x + 3y = -4$
   $6x - 2y = 7$

Which method will you use and why?

Simultaneous Equations

Solutions on the next slide….

Simultaneous Equations Solutions

1. $2x + y = 7$
   $2x - y = 1$

→ Eliminate the y terms by adding the two equations

$4x = 8$

$x = 2$

Sub $x = 2$ into either equation to find y

$2 \times 2 + y = 7$

$y = 3$

2. $3x + 2y = 7$
   $3x + 5y = 4$

→ Eliminate the x terms by subtracting the second equation from the first

$-3y = 3$

$y = -1$

Sub $y = -1$ into either equation to find y

$3x + 2 \times -1 = 7$

$x = 3$

You can always check your answer is correct by substituting into the other equation and check it works in that one too

Unsure about any of these? Search Solving linear simultaneous equations.

Simultaneous Equations Solutions

3. $y = 4x + 3$
$3x + 2y = 28$

Subsitute the first equation into the second

$3x + 2(4x + 3) = 28$

$11x + 6 = 28$

$x = 2$

Sub $x = 2$ into either equation to find y

$y = 4 \times 2 + 3$

$y = 11$

4. $4x + 3y = -4 \quad \times 2$
$6x - 2y = 7 \quad \times 3$

Eliminate the y terms by adding the equations together

$26x = 13$

$x = \frac{1}{2}$

Subs $x = \frac{1}{2}$ into either equation to find x

$4 \times \frac{1}{2} + 3y = -4$

$y = -2$

You can always check your answer is correct by substituting into the other equation and check it works in that one too

Unsure about any of these? Search Solving linear simultaneous equations.

Maths at the movies

Maths movie makes millions!

“Our latest movie ‘Sum-body loves you’ has generated £15 million in online sales and rentals in the first week of it being released” Simultaneous Studios said at the weekend.

We are unable to tell you how much of that total represents the £6 digital rental versus the £15 cost of purchasing the movie. But we do know there were 1 945 000 transactions overall.

Is there really no way of finding out how many rentals and how many sales there were of the film?

- Use what you have learnt so far to calculate how many individual rentals and sales there were of ‘Sum-body loves you’

Maths at the movies Solution

Maths movie makes millions!

“Our latest movie ‘Sum-body loves you’ has generated £15 million in online sales and rentals in the first week of it being released” Simultaneous Studios said at the weekend.

We are unable to tell you how much of that total represents the £6 digital rental versus the £15 cost of purchasing the movie. But we do know there were 1 945 000 transactions overall.

Let:

x be the number of rentals

y be the number of sales

then

$$x + y = 1\,945\,000$$

and

$$6x + 15y = 15\,000\,000$$

Multiply the first equation by 6 to get:

$$6x + 6y = 11\,670\,000$$

Eliminate x by subtracting this new equation from the second equation:

$$9y = 3\,330\,000$$

So $y = 370\,000$

and $x = 1\,575\,000$

Taxi!

There are two taxi companies

Initial Charge: $£x$
then
£1 per mile

Initial Charge: $£2x$
then
80p per mile

They both charge £12 for a journey of the same distance.

- What is the distance?
- What is the value of x?

Taxi! Solution

There are two taxi companies

Initial Charge: £x
then
£1 per mile

Initial Charge: £$2x$
then
80p per mile

They both charge £12 for a journey of the same distance.

- What is the distance?
- What is the value of x?

Let:
number of miles = n
then

$$x + n = 12$$

and

$$2x + 0.8n = 12$$

Multiply the first equation by 2 to get:

$$2x + 2n = 24$$

Eliminate x by subtracting the second equation from it

$$1.2n = 12$$

$$n = \frac{12}{1.2}$$

$$n = 10$$

$$x = 2$$

- The distance is 12 miles
- The value of x is £2

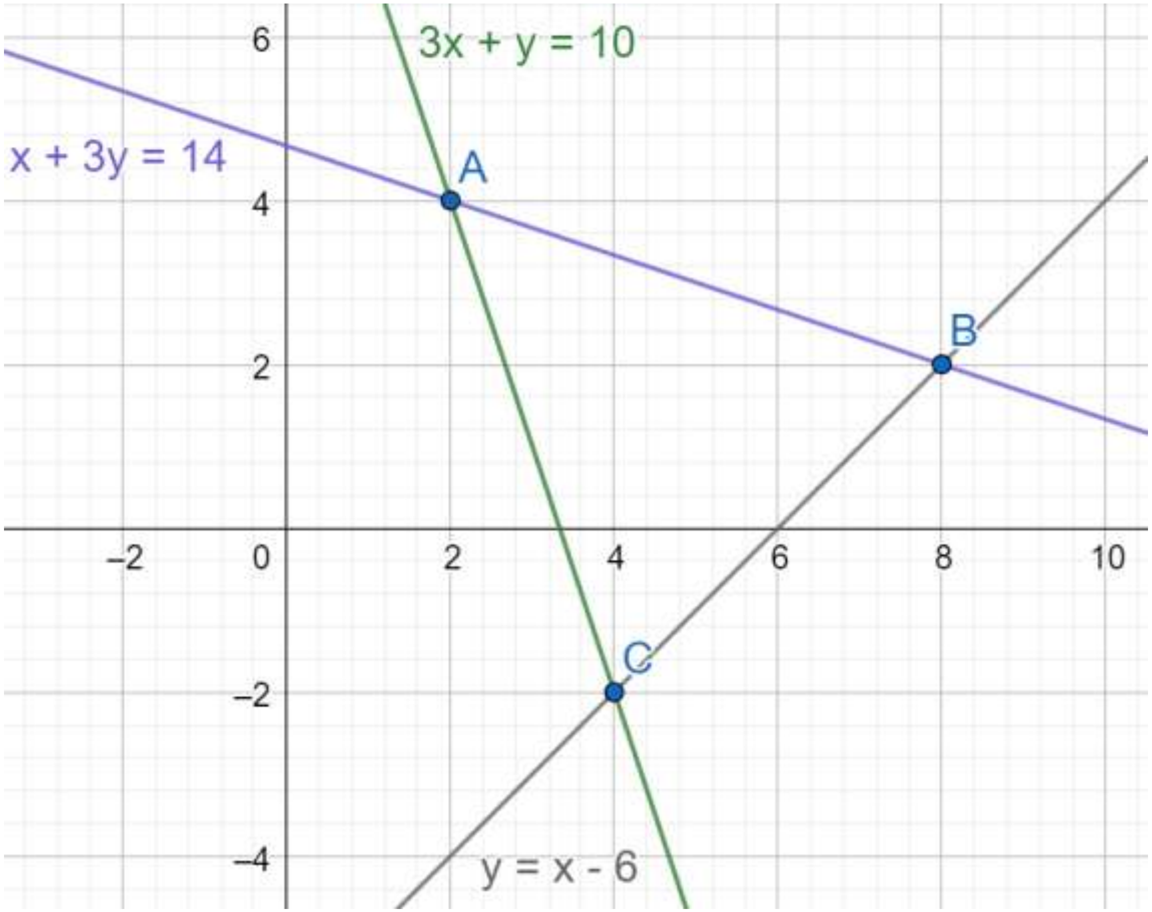
Solving Graphically

Use the graphs to solve these pairs of equations

1. $3x + y = 10$
   $x + 3y = 14$

2. $y = x - 6$
   $3x + y = 10$

3. $x + 3y = 14$
   $y = x - 6$

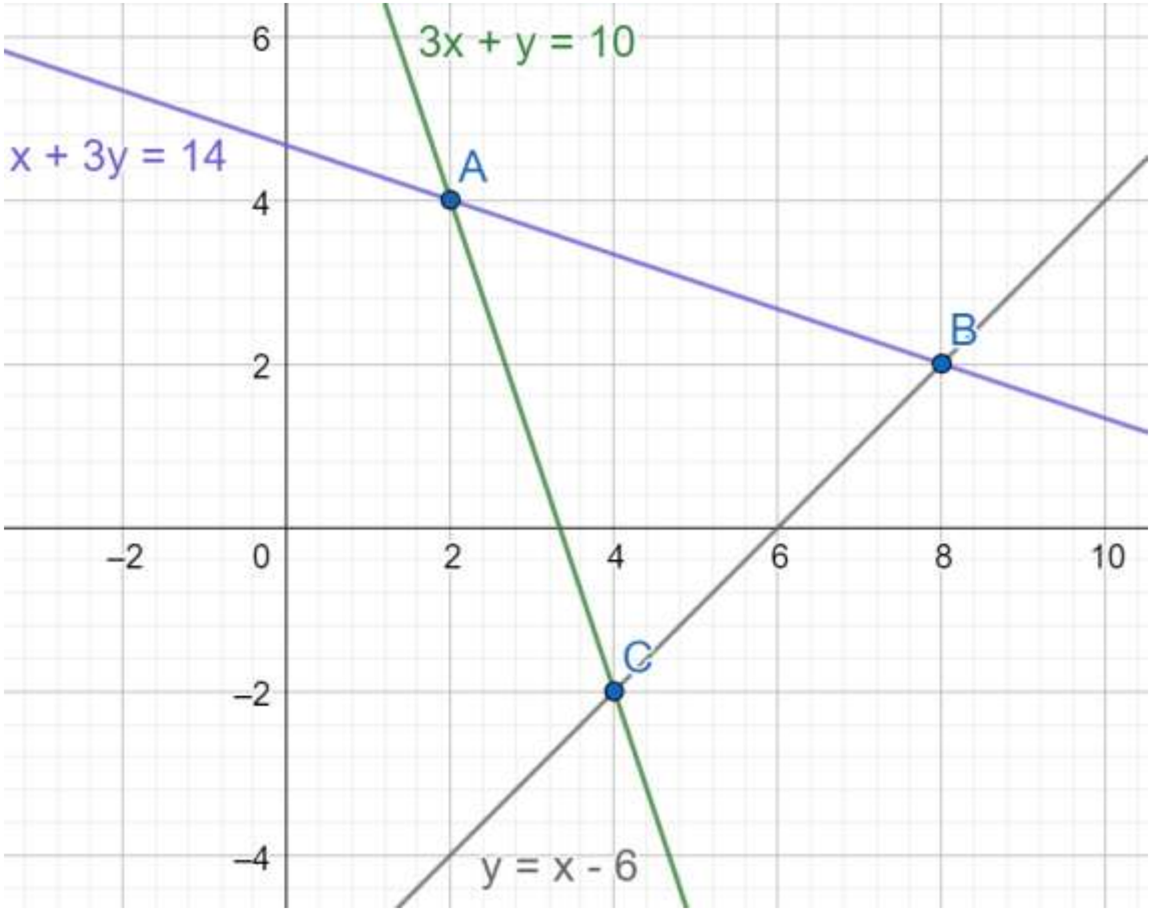


Solving Graphically Solutions

1. $$3x + y = 10$$
$$x + 3y = 14$$
Point A $(2,4)$
$$x = 2, y = 4$$

2. $$y = x - 6$$
$$3x + y = 10$$
Point C $(4, -2)$
$$x = 4, y = -2$$

3. $$x + 3y = 14$$
$$y = x - 6$$
Point B $(8,2)$
$$x = 8, y = 2$$



When we have two equations, in x and y, the solution represents the point where the two lines meet.

Puzzle to ponder

Can you explain algebraically why there are no solutions to the simultaneous equations

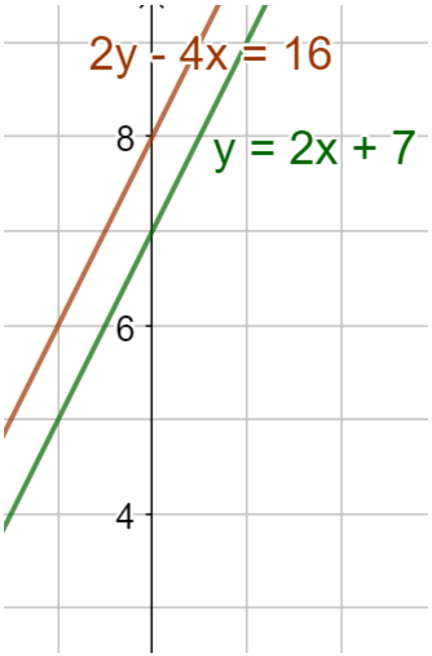
$$y = 2x + 7$$
$$2y - 4x = 16$$

Puzzle to ponder Solution

Can you explain algebraically why there are no solutions to the simultaneous equations

$$y = 2x + 7$$
$$2y - 4x = 16$$

- Rearrange the second equation
- So $y = 2x + 8$
- Both graphs have a gradient of 2.
- The lines are parallel
- Therefore they will never meet.



Solve

$$5x + 3y + z = 24$$
$$4y + 2z = 16$$
$$3z = 18$$

Which equation is the most useful to solve first?

Triple Simultaneous Equations Solution

$$5x + 3y + z = 24$$
$$4y + 2z = 16$$
$$3z = 18$$

What would a picture (or graphs) of this situation look like?

- Dividing the third equation by 3 gives
$$z = 6$$
- Substituting $z = 6$ into the second equation gives
$$4y + 12 = 16$$
So $y = 1$
- Now, substitute $z = 6$ and $y = 1$ into the first equation to get
$$5x + 3 + 6 = 24, \text{ so } x = 3$$
- So the solution is $x = 3, y = 1, z = 6$

Mean Problem

x, y and z satisfy

$$x + y + 3z = 121$$
$$x + 3y + z = 678$$
$$3x + y + z = 356$$

Find the mean of x, y, z, without using a calculator

Mean Problem

x, y and z satisfy

$$x + y + 3z = 121$$
$$x + 3y + z = 678$$
$$3x + y + z = 356$$

Find the mean of x, y, z, without using a calculator

- Write an expression for the mean of x, y, z
- Do you need to find x, y, z seperately to find the mean?

Mean Problem Solution

x, y, z satisfy

$$x + y + 3z = 121$$
$$x + 3y + z = 678$$
$$3x + y + z = 356$$

Find the mean of x, y, z, without using a calculator

- The mean of x, y, z is $\frac{x+y+z}{3}$
- Adding the three equations together gives
- $5x + 5y + 5z = 1155$
- Dividing by 5 we get
- $x + y + z = 231$
- Finally divide by 3 to get the mean = $\frac{x+y+z}{3} = 77$

Still want more?

Read how solving linear equations is an important part of many jobs – including those involving computer graphics, economics and genetics.

Discover the type of maths that is used when making blockbuster movies and how to do it.

Watch this animated history of operational research about its origins in the first and second world wars - when maths was used not only to improve operations but to save lives!

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