Advanced Mathematics Support Programme®

Did you know?

Gradients can be represented in different ways.

You will have seen road signs warning road users of steep hills, but what do the measurements mean?

A gradient of $1:5$ means for every 5m you travel horizontally you travel 1m vertically.

A gradient of 16% means that the vertical distance travelled is 16% of the horizontal distance.

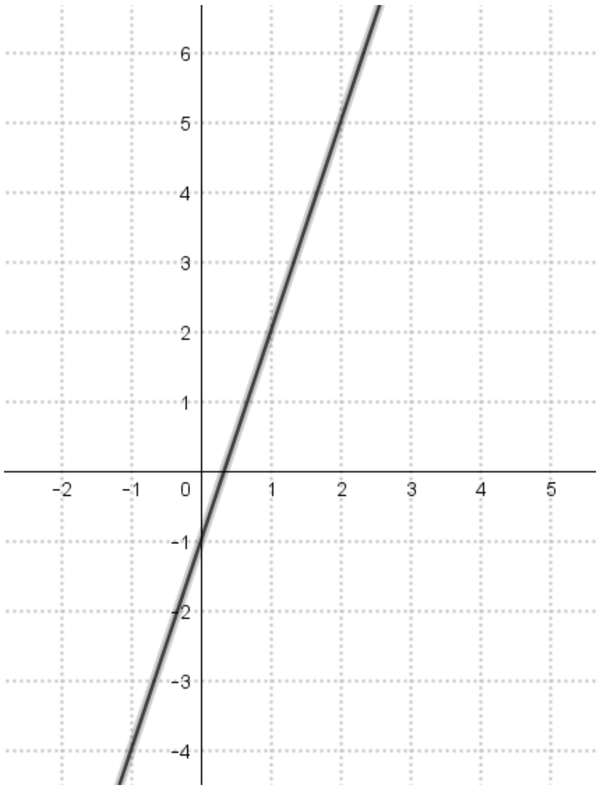
So for every 100m across you go 16m up.

Where is the steepest street in the world?

This has caused real controversy in the last year – find out why here
They hold a Jaffa rolling contest down the street every year!

Straight Line Graphs 1

1. What are the gradient and intercept of the line $y = 3x - 5$
2. Find the gradient of the line connecting (3,10) and (1,6)
3. Find the midpoint between the points (3,-8) and (-1,4)
4. Find the distance between points (1,10) and (4,18)
5. What is the equation of the line with gradient 3 that passes through (5,8)?
6. Does the line $y = 2x - 3$ pass through (1,-1)? Explain how you know.
7. Find the equation of a line that is parallel to $y = 5x - 2$ that passes through (2,19)
8. What is the equation of this graph?



Straight Line Graphs 1

Solutions on the next slide….

Straight Line Graphs 1 - Solutions

1. What are the gradient and y intercept of the line $y = 3x - 5$

Gradient = 3, intercept = -5

2. Find the gradient of the line connecting (3,10) and (1,6)

Gradient = $\frac{10-6}{3-1} = 2$

Note: $\frac{6-10}{1-3} = -\frac{4}{-2} = 2$ gives the same answer

3. Find the midpoint between the points (3,-8) and (-1,4)

Midpoint = $(\frac{3+-1}{2}, \frac{-8+4}{2}) = (1, -2)$

4. Find the distance between the points (1,10) and (4,18)

Distance = $\sqrt{(4-1)^2 + (18-10)^2}$
$= \sqrt{73}$

Straight Line Graphs 1 - Solutions

5. What is the equation of the line with gradient 3 that passes through (5,8)?

$y = 3x + c$
Using our coordinate (5,8)
$8 = 3 \times 5 + c$ so $c = 8 - 15 = -7$
Equation is $y = 3x - 7$

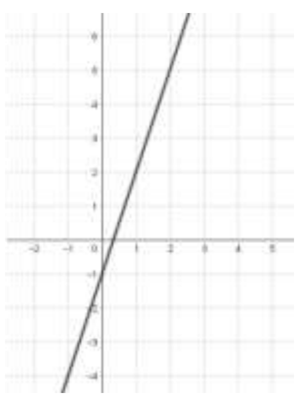
6. Does $y = 2x - 3$ pass through (1,-1)? Explain how you know.

Substituting $x = 1$,
$y = 2 \times 1 - 3 = -1$
Yes the line passes through (1,-1)

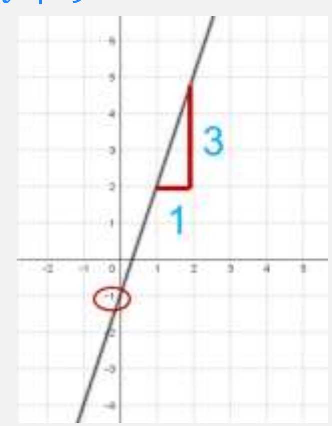
7. Find the equation of a line that is parallel to $y = 5x - 2$ that passes through (2,19)

$y = 5x + c$
Using our coordinate (2,19)
$19 = 5 \times 2 + c$ so $c = 19 - 10 = 9$
Equation is $y = 5x + 9$

8. What is the equation of this graph?



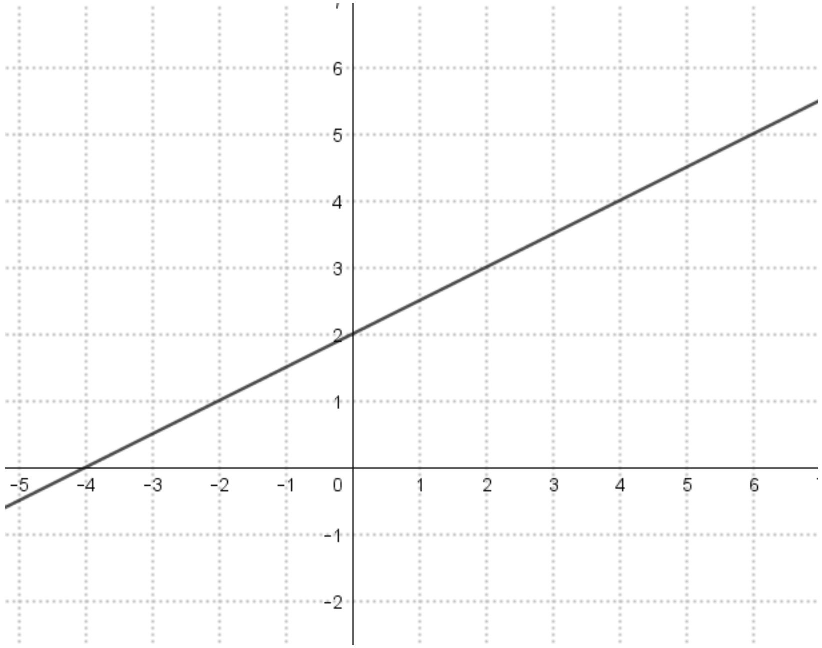
$$y = 3x - 1$$



Unsure about any of these? Search Straight Line Graphs. Next try Straight Line Graphs 2….

Straight Line Graphs 2

1. What are the gradient and y intercept of the line $y = 2x - 7$
2. Find the gradient of the line connecting (1,4) and (-1,0)
3. Find the midpoint between the points (-2,10) and (6,4)
4. Find the distance between the points (4,11) and (-1,15)
5. What is the equation of the line with gradient 2 that passes through (1,4)?
6. Does the line $y = -2x + 5$ pass through (3,1)? Explain how you know.
7. Find the equation of a line that is parallel to $y = -\frac{3}{2}x - 1$ that passes through (6,4)
8. What's the equation of this graph?



Straight Line Graphs 2

Solutions on the next slide….

Straight Line Graphs 2 - Solutions

1. What are the gradient and y intercept of the line $y = 2x - 7$

→ Gradient = 2, intercept = -7

2. Find the gradient of the line connecting (1,4) and (-1,0)

→ Gradient = $\frac{4-0}{1--1} = 2$

3. Find the midpoint between the points (-2,10) and (6,4)

→ Midpoint = $(\frac{-2+6}{2}, \frac{10+4}{2}) = (2,7)$

4. Find the distance between the points (4,11) and (-1,15)

→ Distance = $\sqrt{(4--1)^2 + (11-15)^2}$

$= \sqrt{41}$

Straight Line Graphs 2 - Solutions

5. What is the equation of the line with gradient 2 that passes through (1,4)?

$y = 2x + c$

Using our coordinate (1,4)

$4 = 2 \times 1 + c$ so $c = 4 - 2 = 2$

Equation is $y = 2x + 2$

6. Does the line $y = -2x + 5$ pass through (3,1)? Explain how you know.

Substituting $x = 3,\ y = 3 \times -2 + 5 = -1$

No, the line doesn't pass through (3,1) as when $x = 3, y = -1$

It passes through (3,-1)

7. Find the equation of a line that is parallel to $y = -\frac{3}{2}x - 1$ that passes through (6,4)

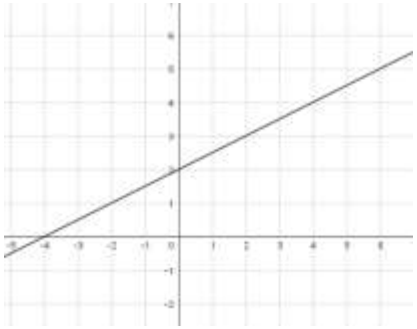
$y = -\frac{3}{2}x + c$

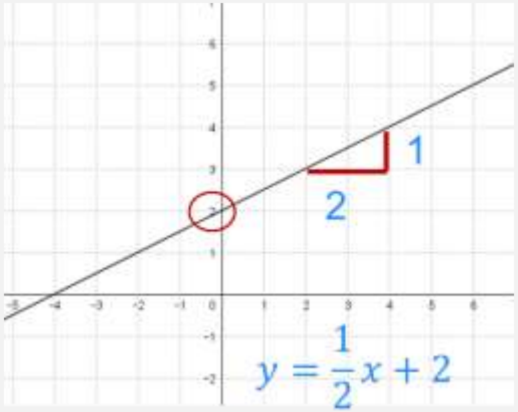
Using our coordinate (6,4)

$4 = -\frac{3}{2} \times 6 + c$ so $c = 4 + 9 = 13$

Equation is $y = -\frac{3}{2}x + 13$

8. What's the equation of this graph?





Which Form?

The same graph can be described using either of these two forms of the equation

$$y = -2x + 2$$
$$2x + y = 2$$

- Which of the two equations do you prefer?
- Which equation would you feel confident in sketching the graph from?

Which Form?

Most students are more comfortable with $y = -2x + 2$

They can then use the gradient and intercept to help them sketch the graph

If you were given the equation as $2x + y = 2$ you could rearrange to get $y = -2x + 2$

Or you could find the x and y intercepts like this:

Use the fact that $y = 0$ when the line crosses the x axis.

This means we can substitute $y = 0$ into

$$2x + y = 2$$
$$2x + 0 = 2$$
$$2x = 2$$
$$x = 1$$

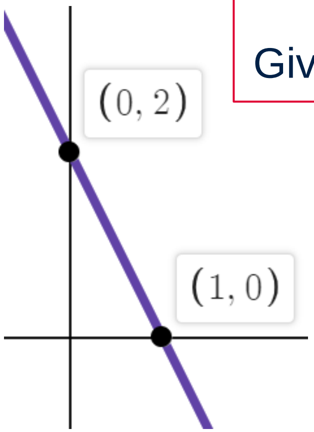
Giving the x intercept as $(1,0)$

Use the fact that $x = 0$ when the line crosses the y axis.

This means we can substitute 0 for x into

$$2x + y = 2$$
$$0 + y = 2$$
$$y = 2$$

Giving the y intercept as $(2,0)$



Do they cross?

Line A passes through the points $(-3,1)$ and $(3,5)$

Line B passes through the points $(0,-4)$ and $(6,4)$

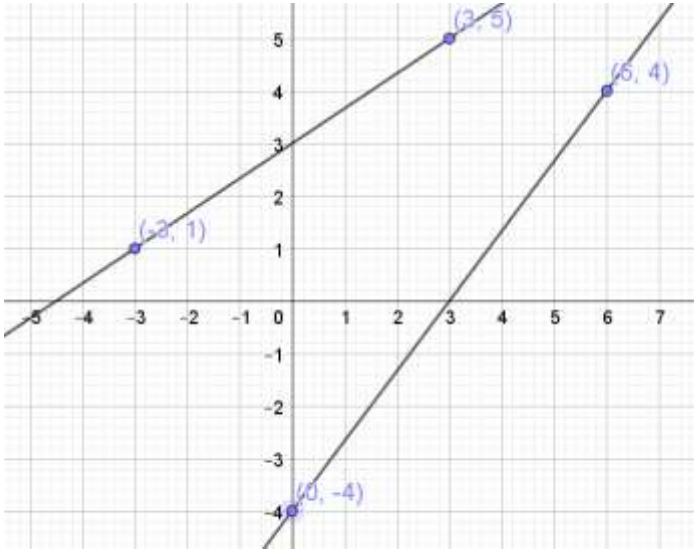
- By sketching can you tell if the lines will meet?
- If they do meet what the points of intersection?

Do they cross ?

Solutions on the next slide….

Do they cross? Solution

From a sketch we can see that the lines are not parallel and will meet at some point



Fancy a challenge?

- Can you find where the lines will meet using algebra

Do they cross? Challenge Solution

- Can you find where the lines will meet using algebra

Find the gradient of the line between the points.
Then substitute in the corresponding x and y values from one of the co-ordinates, along with m, into $y = mx + c$ to find c

The equations of the lines are

$$y = \frac{4}{3}x - 4$$

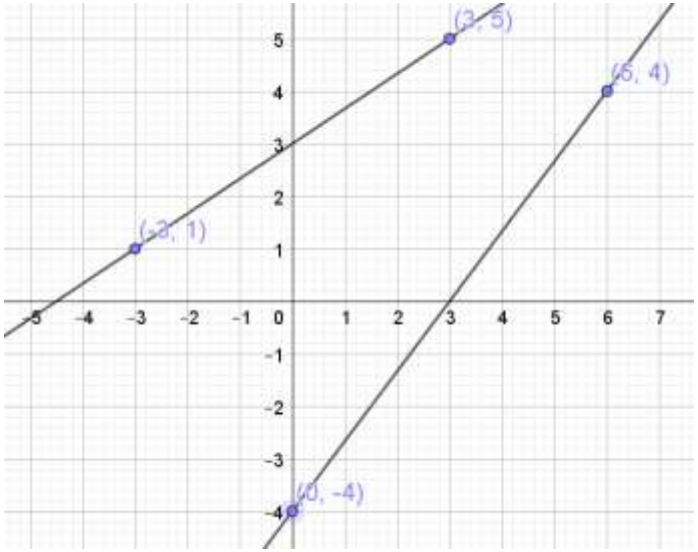
Or rearrange to get

$$4x - 3y = 12$$

$$y = \frac{2}{3}x + 3$$

Or rearrange to get

$$2x - 3y = -9$$



Now solve the simultaneous equations (the elimination method works well here) to find where the lines meet.

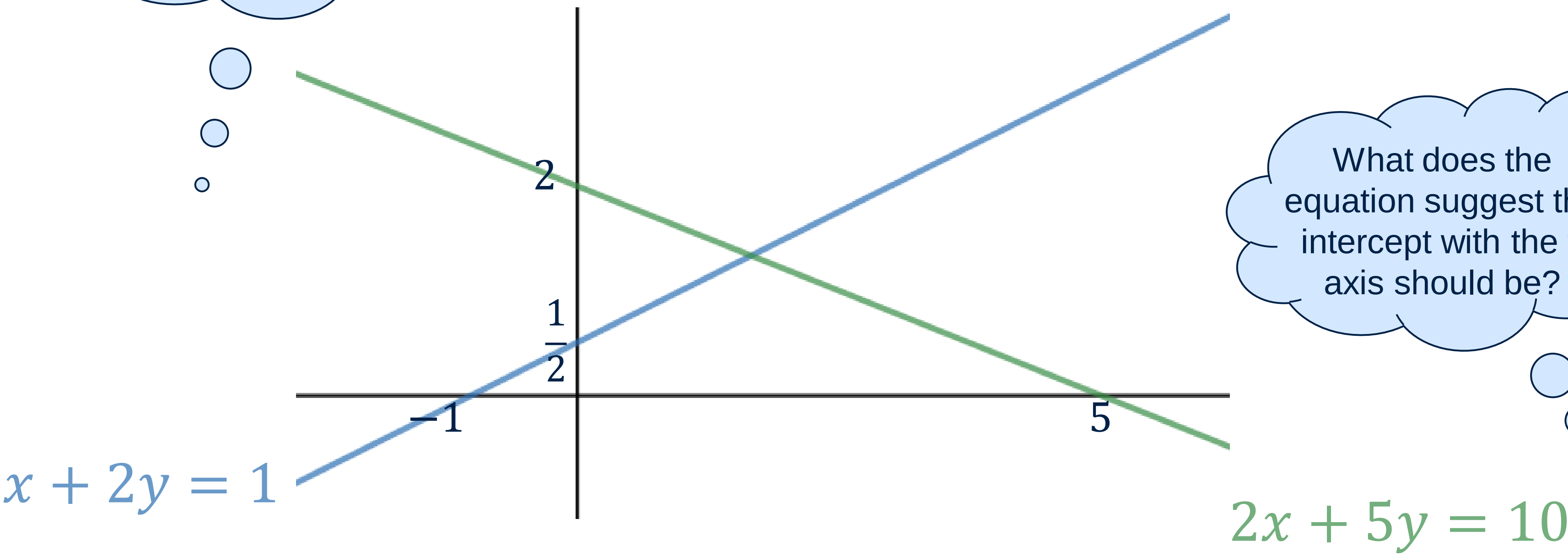
The lines intersect at $(10.5, 10)$

Picture this

- Is this an accurate sketch of these two lines?

Think about whether the equation suggests a positive or negative gradient

What does the equation suggest the y intercept with the y axis should be?



Picture this

Solutions on the next slide….

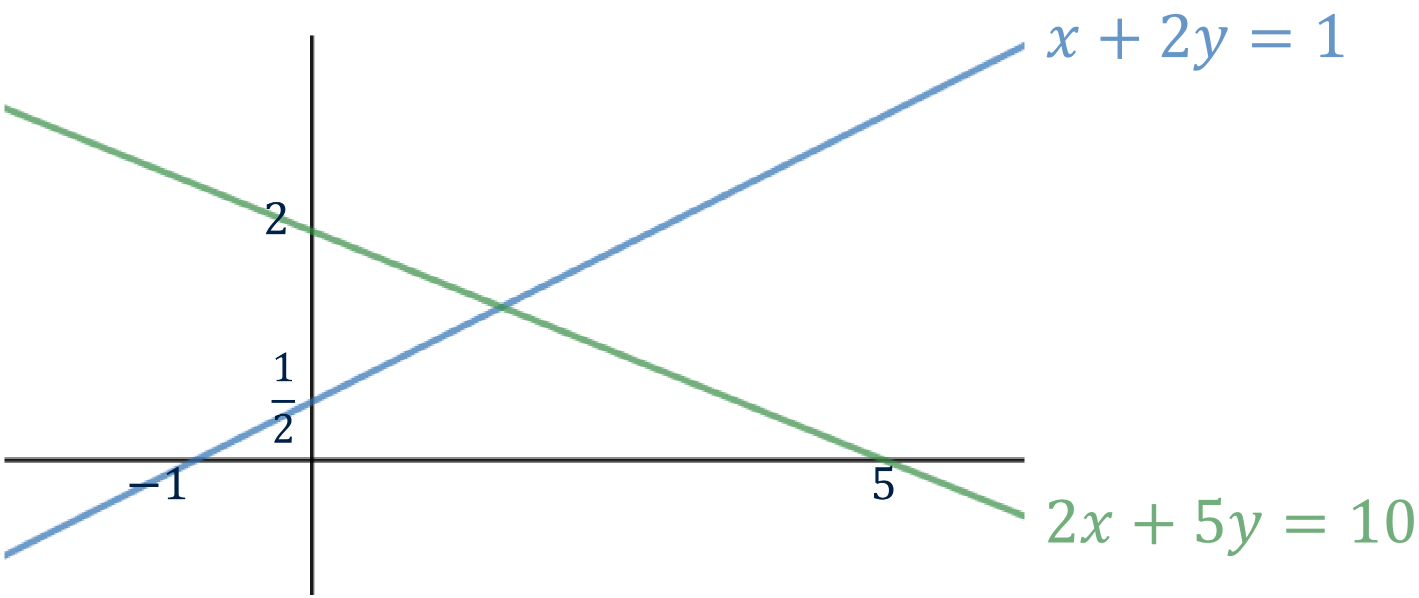
Picture this Solution

- Is this an accurate sketch of these two lines?

$x + 2y = 1$ should have a negative gradient, which it doesn't in the sketch

Also, the y intercept is $\left(0, \frac{1}{2}\right)$, the x intercept is $\left(\frac{1}{1}, 0\right) = (1,0)$

So they have sketched $-x + 2y = 1$



$2x + 5y = 10$ should have a negative gradient, which it does.

The y intercept is $\left(0, \frac{10}{5}\right) = (0,2)$ and the x intercept is $\left(\frac{10}{2}, 0\right) = (5,0)$

So this line is correct

The plot thickens…

Complete the information in the table for each equation below:

- Find the co-ordinates of the x and y intercepts
- Decide if the gradient of the graph would be positive or negative

Using the information from the table, sketch all the graphs on one set of axes to find:

- A pair of lines that are parallel
- A pair of lines that are perpendicular
- A pair of lines that intersect at $(-2, 2)$

Name	Equation	x-intercept	y intercept	Positive/negative gradient
A	$y - 2x - 1 = 0$			
B	$y = 3$			
C	$3x + 4y = 2$			
D	$2x - y + 6 = 0$			
E	$2y + x = 4$			
F	$2x + y - 3 = 0$			

The plot thickens…

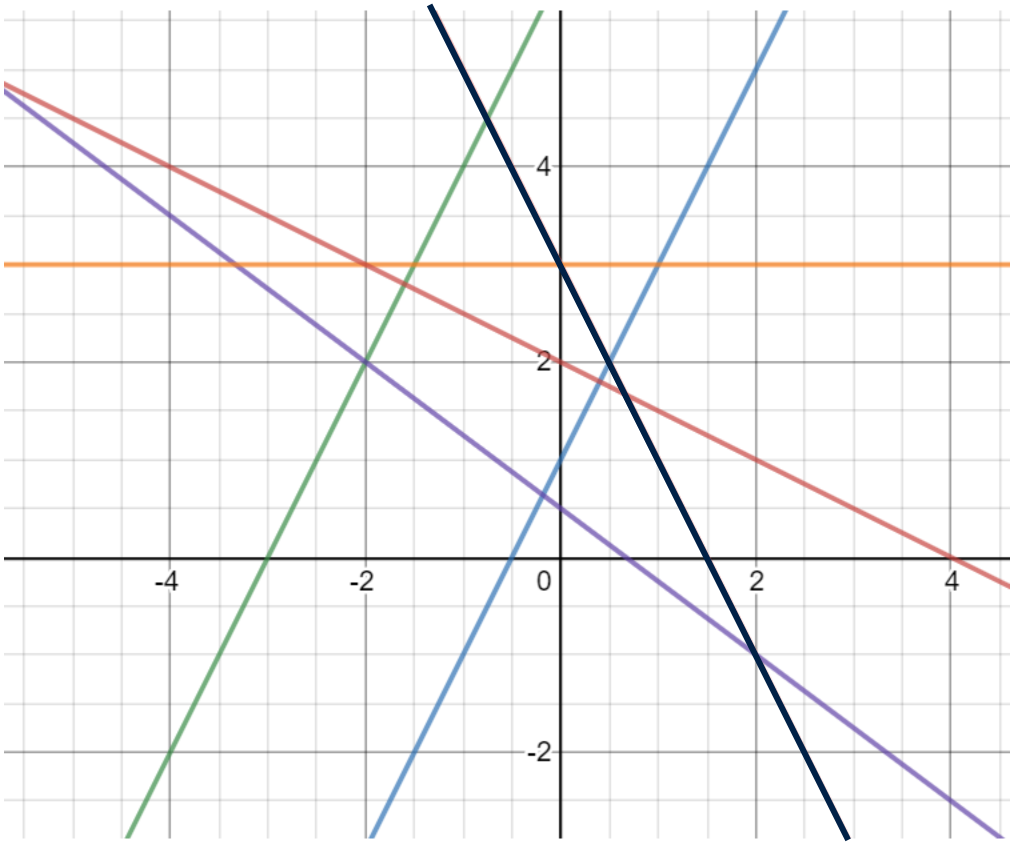
Solutions on the next slide….

The plot thickens…Solution

Name	Equation	x-intercept	y intercept	Positive/negative gradient
A	$y - 2x - 1 = 0$	$(-\frac{1}{2}, 0)$	$(0, 1)$	Positive
B	$y = 3$	No intercept	$(0, 3)$	Horizontal line
C	$3x + 4y = 2$	$(\frac{2}{3}, 0)$	$(0, \frac{1}{2})$	Negative
D	$2x - y + 6 = 0$	$(-3, 0)$	$(0, 6)$	Positive
E	$2y + x = 4$	$(4, 0)$	$(0, 2)$	Negative
F	$2x + y - 3 = 0$	$(\frac{3}{2}, 0)$	$(0, 3)$	Negative

The sketches of all the graphs are on the next page

The plot thickens… Solution



A	$y-2x-1=0$
B	$y=3$
C	$3x+4y=2$
D	$2x-y+6=0$
E	$2y+x=4$
F	$2x+y-3=0$

Did you find?

- A pair of equations that do not intersect — A and D as they are parallel
- A pair of equations that are perpendicular — A and E or D and E
- A pair of equations that intersect at $(-2,2)$ — C and D intersect at (-2,2)

It is possible to find all the intersections of the lines – which ones are more easily found using algebra?

Two geometry problems

DEF is an isosceles right angled triangle

The line passing through D and F has the equation

$$x + 3y = 15$$

D is the co-ordinate $(6,3)$
E is the co-ordinate $(5,0)$

The angle EDF is the right angle

Can you find:
- The equation of line DE?
- The possible coordinates of F?
- The equation of line EF?

Hint: Sketch the graphs!!

ABCD is a parallelogram

The line passing through C and D has the equation $y = 7$

The line CD is 5 units long

D has coordinate $(2,7)$

C has both positive x and y co-ordinates

The line through AC has equation

$$3x + 2y = 35$$

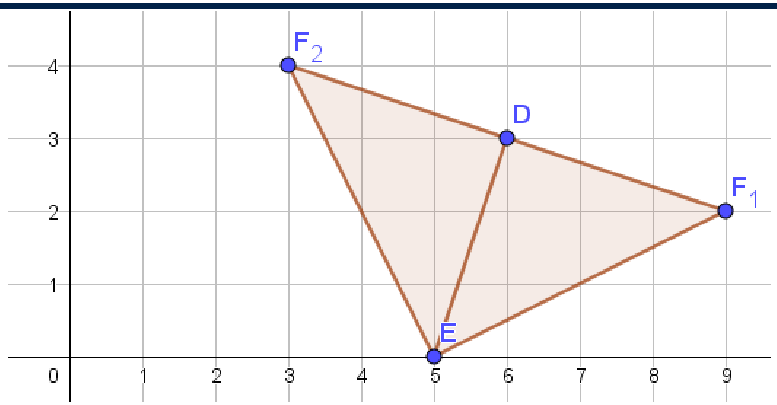
A has coordinate $(9,4)$

Can you find:
- The coordinate of C?
- The equation of line AB?
- The equation of line BD?
- The area of the parallelogram?

Two geometry problems

Solutions on the next slide….

Two geometry problems Solutions



DEF is an isosceles right angled triangle
The line passing through D and F has the equation

$$x + 3y = 15$$

D is the co-ordinate $(6,3)$
E is the co-ordinate $(5,0)$
The angle EDF is the right angle

Can you find:

- The equation of line DE? $y = 3x - 15$

The gradient of DE is $\frac{3-0}{6-5} = 3$ There are different ways to find $c = 15$. e.g. continuing the line DE and observing where it crosses the y axis or substituting D (6,3) into $y = mx + c$

- The possible coordinates of F? $(3,4)$ *or* $(9,2)$

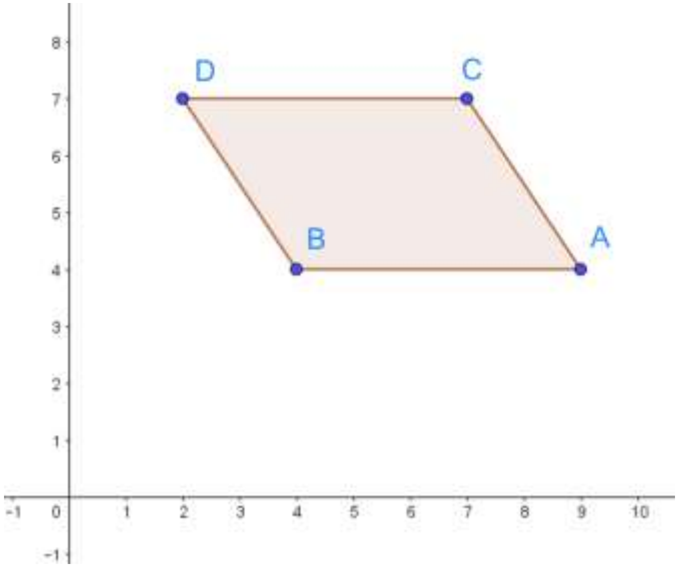
From your sketch you should see there are two possible co-ordinates that F could be. As the triangle is isosceles the length of DF=DE= $\sqrt{3^2 + 1^2} = \sqrt{10}$ by Pythagoras' theorem

- The equation of line EF? $y = \frac{1}{2}x - \frac{5}{2}$ or $y = -2x + 10$

Gradient of $EF_1 = \frac{2-0}{9-5} = \frac{1}{2}$, then substitute (5,0) into $y = \frac{1}{2}x + c$ to find c

Gradient of $EF_2 = \frac{4-0}{3-5} = -2$, then substitute (5,0) into $y = -2x + c$ to find c

Two geometry problems Solutions



ABCD is a parallelogram
The line passing through C and D has the equation

$$y = 7$$

The line CD is 5 units long
D has co-ordinate $(2,7)$
C has both positive x and y co-ordinates
The line through AC has equation

$$3x + 2y = 35$$

A has co-ordinate $(9,4)$

Can you find:

- The coordinate of C? $(7,7)$

  It lies on the line $y = 7$ and we know that CD has length 5 so C must be (7,7)
- The equation of line AB? $y = 4$

  AB is parallel to CD and A has co-ordinate (9,4)
- The equation of line BD? $y = -\frac{3}{2}x + 10$

  From your drawing

  BD is parallel to AC, so by rearranging $m = -\frac{3}{2}$ Substituting (4,4) into $y = -\frac{3}{2}x + c$ gives $c = 10$
- The area of the parallelogram? 15 units^2

  Area=base × perpendicular height = 5×3

Geometry from equations

The following equations enclose a square:

$$y - 2 = x$$
$$y + x = 6$$
$$y = x - 1$$
$$y + x - 3 = 0$$

- Which are the two pairs of parallel sides?
- What are the coordinates of all 4 vertices
- How can you convince yourself this is a square?

This task is inspired by https://undergroundmathematics.org/geometry-of-equations/simultaneous-squares

Fancy a challenge? Then give that task a go! It's tricky but fun and only uses GCSE Maths skills.

Geometry from equations

Solutions on the next slide….

Geometry from equations Solutions

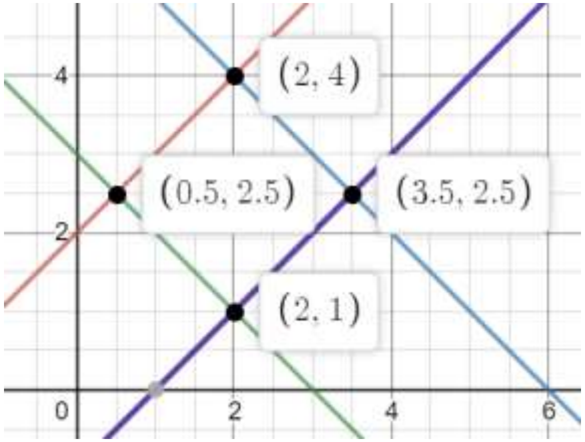
- Which are the two pairs of parallel sides?

If we rearrange the 4 equations to get:

$$y = x + 2 \quad (1)$$
$$y = -x + 6 \quad (2)$$
$$y = x - 1 \quad (3)$$
$$y = -x + 3 \quad (4)$$

We can see that equations (1) and (3) are parallel as are (2) and (4)

- What are the coordinates of all 4 vertices?



- How can you convince yourself it's a square?

As well as all the lines that meet being perpendicular, you also need to show they all have the same length. You can do this by using Pythagoras' theorem, or calculating the column vector.

Sketching Linear Inequalities

- Sketch and shade the following inequalities.

1. $y \leq 6$

2. $x < 6$

3. $x + 2y \geq 8$

4. $3x + 2y \geq 12$

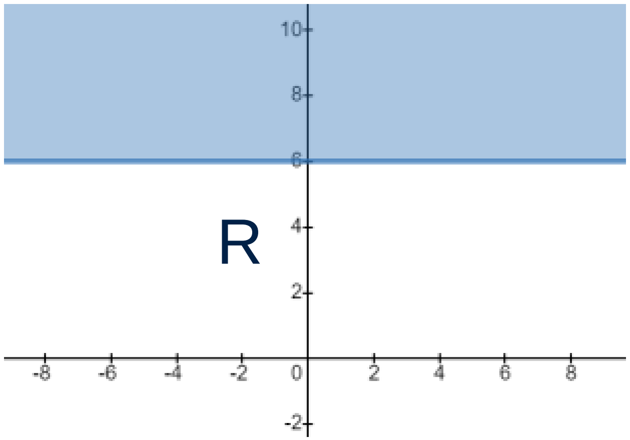
- Shade out the side of the line that doesn't satisfy the inequality.
- Label the correct region **R**

Sketching Linear Inequalities

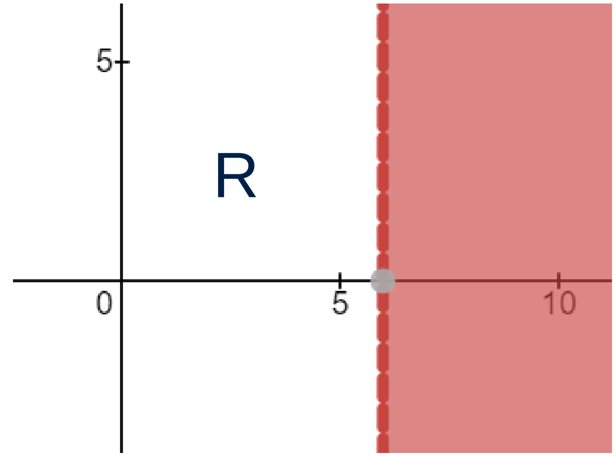
Solutions on the next slide….

Sketching Linear Inequalities Solution

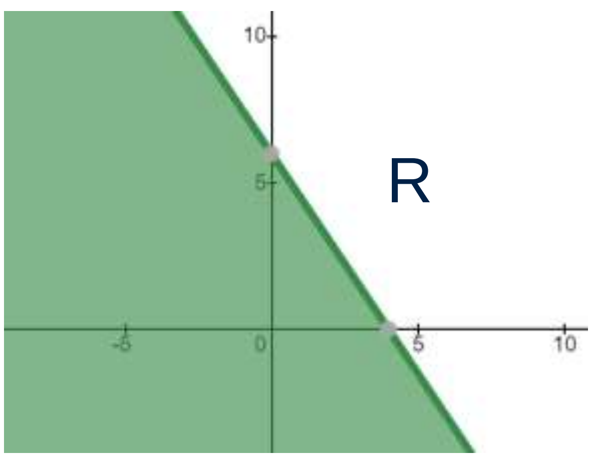
1. $y \leq 6$



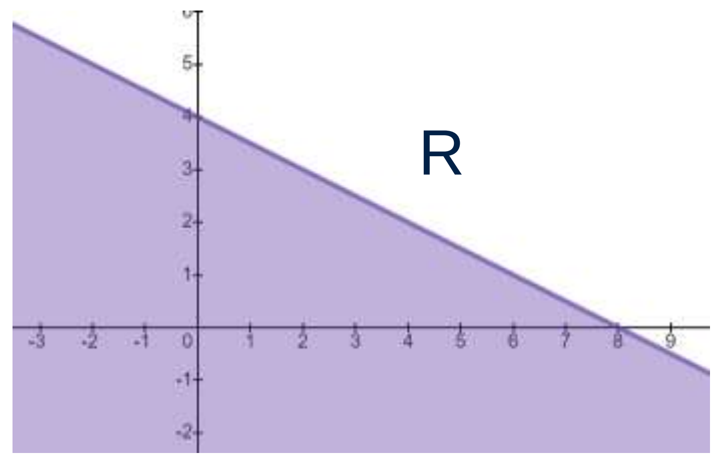
2. $x < 6$



3. $x + 2y \geq 8$



4. $3x + 2y \geq 12$

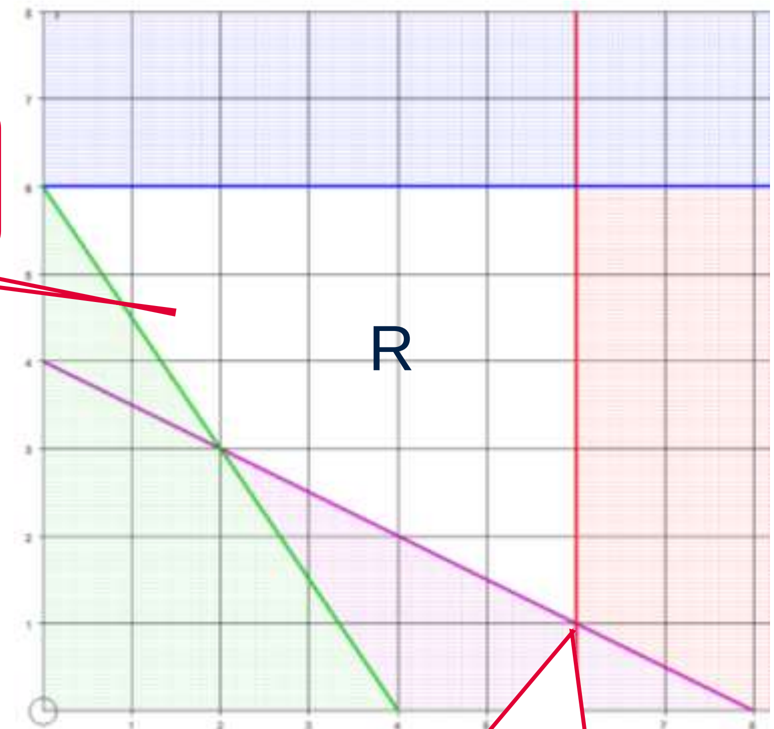


Try drawing these and other inequalities on a graphing package such as Desmos or GeoGebra

Linear Programming

Below is a graph that shows the feasible region R satisfied by the all inequalities from the previous slide.

R, the unshaded region is called the FEASIBLE REGION. Points in this region satisfy all of the inequalities.



In Linear Programming linear inequalities are used to find solutions to real life problems.

The 'optimal' or best solution for is found for a particular objective.

The feasible region has four vertices (corner points). What are the coordinates?

- Use the diagram above to have a go at this question

Maximise the value of $x + y$ within the region satisfied by the inequalities:

$$x + 2y \geq 8, 3x + 2y \geq 12, y \leq 6, x \leq 6$$

Linear Programming Solution

Maximise the value of $x + y$ within the region satisfied by the inequalities:

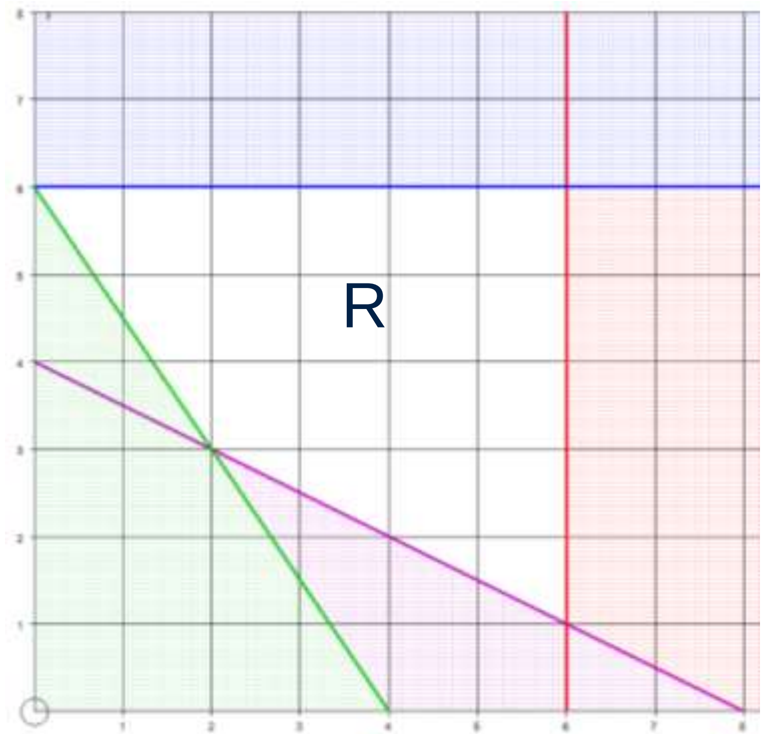
$$x + 2y \geq 8, 3x + 2y \geq 12, y \leq 6, x \leq 6$$

To maximise the value of $x + y$ within the feasible region, we substitute in the coordinates of each vertex.

$$\begin{array}{ll} (0,6) & x + y = 0 + 6 = 6 \\ (2,3) & x + y = 2 + 3 = 5 \\ (6,6) & x + y = 6 + 6 = 12 \\ (6,1) & x + y = 6 + 1 = 7 \end{array}$$

So the maximum value of $x + y$ is 12 at the point (6,6)

You can check that other points within the feasible region give values of $x + y$ that are less than 12



Click

To learn more about linear programming and see a real life question

To try out some linear programming for yourself – with solutions here!

Catching Stars

Click here to try a Linear Marbleslides Challenge

You will be investigating the features of linear graphs whilst trying to catch as many stars as possible

You can join the activity without signing in or entering your real name.

Still want more?

Read about different ways of representing straight lines. Some of these representations you will come across at A Level and some offer an insight to mathematics studied at a higher level.

Discover how electronics can help with graphical linear algebra as it is actually based on circuit diagrams!

Watch how this robot creates curved art using only straight lines. Why not have a go yourself?

Contact the AMSP

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Advanced_Maths